# Electrical Energy Systems
## (Power Applications of Electricity)

**Summary of selected topics from University of Washington course EE 351: Energy Systems**
**taught Fall 2015 by Prof. Baosen Zhang (BXZ)**
**compiled by Michael C. McGoodwin (MCM). Content last updated 5/17/2016**

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# Introduction

Electrical Energy Systems is a large and very important subject—these systems permeate our advanced civilizations and we would regress to the 17th Century without them. The subject matter is complex and hard to set down into a document (especially circuit diagrams), and vita brevis, so as usual I have been quite selective in what I have chosen to include here. My emphasis has been on topics that are/were:

- personally relevant and practical (such as household electrical configurations)
- interesting or not personally well understood in concepts or terminology (such as 3-phase systems)
- of current societal interest (such as renewable energy resources)

I merely audited this course, and it was the first engineering course ever attended (my major was Physics many decades earlier). I therefore claim no expertise and assume that this summary contains errors, including errors that, if implemented, might lead to a shock hazard! I create summaries like this mostly to assist my own learning process broadly interpreted, to provide a convenient and semi-permanent record of what I studied for future reference, and secondarily to help students and others wanting to explore these topics.

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Suggestions and corrections would be graciously accepted. Send email to this address (reformatted):
MCM at McGoodwin period NET

The course syllabus includes the following:

**Instructor**: Baosen Zhang, zhangbao@uw.edu
Assistant professor

Graduate PhD from Berkeley in 2013
Undergrad from University of Toronto
Research in Power systems, smart grids, cyberphysical systems with people in the loop

**TAs**:

1) John J. Sealy, sealyj2@uw.edu;
2) Daniel Olsen, djolsen@uw.edu

**Textbook**: *Electric Energy: An Introduction*, 3rd Edition, by Mohamed A. El-Sharkawi, CRC Press 2013, hereafter called *EEAI3*. This is an excellent textbook which I have enjoyed reading, and noteworthy in being the work of a UW professor just retired. It could be fruitful reading for a variety of readers. I have found many errors of a minor degree, and may provide a suggested errata.

**Class Location**: Room 037, Electrical Engineering Building (EEB)

**Catalog Description**: "Develops understanding of modern energy systems through theory and analysis of the system and its components. Discussions of generation, transmission, and utilization are complemented by environmental and energy resources topics as well as electromechanical conversion, power electronics, electric safety, renewable energy, and electricity blackouts."

**More Detailed Description**: In this class we will cover the following:

- History of power systems
- Basics components of power systems: Generation, transmission and distribution of electricity
- Renewable energy: Mainly on wind and solar, and how they are different from "conventional energy"
- Electric machines and safety
- After not changing much for decades, why is energy system suddenly a "hot" topic again: e.g., what is a smart grid?

**Labs**:

Three Labs (First lab starting Oct 26th, Orientation Oct 19th). "There is a policy that only students who are taking it for a grade can do the labs." I did not attend these labs.
Instructional Lab is managed by Bill Lynes.

Course Website including Syllabus:
https://canvas.uw.edu/courses/988266 (UW ID login needed)

**Resources for Electrical Engineering**

Electrical Engineering
Get research recommendations and tips tailored to your subject area via this online guide.

Citation Styles & Tools
Find citation style guides, citation management tools, and more.

Engineering Library
The Engineering Library supports research and study in numerous engineering fields as well as computer science.

Electronic Schematic Creation:

- Scheme-It
  Free. Many symbols. Prepares bill of materials (BOM) PRN. Many objects cannot be labeled flexibly.
- Electronic Schematic Creation & Calculations: Circuit-Lab[1]
  Cannot add subscripts to labels; drawing arrows for voltage labeling clumsy; clunky; Requires student registration for specific EE course etc., otherwise costs. Documentation

## Book chapters included in the course {and/or discussed in this summary}

**Bold** = substantial course coverage by instructor Parentheses indicate coverage by MCM

Chap 1: **History of Power Systems** {MCM selections completed}

Chap 2: **Basic Components of Power Systems** {MCM selections completed}

Chap 3: **Energy Resources** {MCM selections completed}

[1] Students at UW are eligible for CircuitLab Student Edition.

Chap 4: **Power Plants** {MCM selections completed}

Chap 5: **Environmental Impact of Power Plants** {MCM selections completed}

Chap 6: **Renewable Energy** [Solar, Wind, Geothermal, Biomass, Hydrokinetic] {Omitted}

Chap 7: **Alternating Current Circuits** {MCM selections completed}

Chap 8: **Three-Phase Systems** {MCM selections completed}

Chap 9: **Electric Safety** {MCM selections completed}

Chap 10: **Power Electronics** {MCM selections completed}

Chap 11: **Transformers** {MCM selections completed}

Chap 12: **Electric Machines** {MCM selections completed}

Chap 13: Power Quality {Not studied in this 300 level class, MCM selections completed}

Chap 14: **Power Grid and Blackouts** {Briefly discussed in class, MCM selections completed}

Chap 15: Future Power Systems {Not studied in class, MCM selections completed}

# History and Basic Science

## Notable Persons in the History of Electricity

This section derives in part from chapter 1.

**Thales of Miletus** (600 BCE): static electricity from amber (ἤλεκτρον =elektron) when rubbed by fur

**William Gilbert (**24 May 1544 – 30 November 1603), book *De Magnete* (1600), originated “electricity”, father of electricity & magnetism.

**Alessandro Giuseppe Antonio Anastasio Volta** (18 February 1745 – 5 March 1827): Voltaic pile. Volt is the SI unit of electric potential.

**Hans Christian Ørsted** (14 August 1777 – 9 March 1851): discovered that electric currents create magnetic fields, deflecting a compass needle. Oersted is the CGS unit of the magnetic field strength (auxiliary magnetic field H).

**André-Marie Ampère** (20 January 1775 – 10 June 1836): French physicist and mathematician who was one of the founders of the science of classical electromagnetism, which he referred to as "electrodynamics". Ampere’s Law. Ampere is the SI unit of current.

**Georg Simon Ohm** (16 March 1789 – 6 July 1854): German physicist and mathematician. Ohm found that there is a direct proportionality between the potential difference (voltage) applied across a conductor and the resultant electric current. This relationship is known as Ohm's law. The ohm is the SI unit of electrical resistance.

**Michael Faraday** (22 September 1791 – 25 August 1867), English scientist who contributed to the fields of electromagnetism and electrochemistry. His main discoveries include those of electromagnetic induction, diamagnetism and electrolysis. Built a device that became the basis for the AC motor. His work inspired James Clerk Maxwell. The derived SI unit of capacitance is the farad.

**James Clerk Maxwell** (13 June 1831 – 5 November 1879): Scottish scientist in the field of mathematical physics. His most notable achievement was to formulate the classical theory of electromagnetic radiation, bringing together for the first time electricity, magnetism, and light as manifestations of the same phenomenon. Maxwell's equations for electromagnetism have been called the "second great unification in physics" after the first one realized by Isaac Newton.

With the publication of A Dynamical Theory of the Electromagnetic Field in 1865, Maxwell demonstrated that electric and magnetic fields travel through space as waves moving at the speed of light.

**Hippolyte Pixii** (1808–1835) , instrument maker from Paris. In 1832 he built an early form of alternating current electrical generator, based on the principle of magnetic induction discovered by Michael Faraday. Pixii's device was a spinning magnet, operated by a hand crank, where the North and South poles passed over a coil with an iron core. ... introducing a commutator, which produced a pulsating direct current.

**Antonio Pacinotti** (17 June 1841 – 24 March 1912) was an Italian physicist, improved DC generator (dynamo) and invented transformer with 2 sets of windings about common core. AC in one winding induced AC in the other.

**John Ambrose Fleming** (1849–1945), English electrical engineer and inventor of the Fleming Valve (thermionic vacuum tube diode). Stated Fleming's left hand rule: When current flows in a wire, and an external magnetic field is applied across that flow, the wire experiences a force perpendicular both to that field and to the direction of the current flow. A left hand can be held … so as to represent three mutually orthogonal axes on the thumb (thrust), first finger (magnetic field) and middle finger (current). The right and left hand are used for generators and motors, respectively.

**Lee de Forest** (August 26, 1873 – June 30, 1961)American inventor, self-described "Father of Radio", and a pioneer in the development of sound-on-film recording used for motion pictures. His most famous invention, in 1906, was the three-element "grid Audion", which, although he had only a limited understanding of how it worked, provided the foundation for the development of vacuum tube technology.

**Julius Edgar Lilienfeld** (April 18, 1882 – August 28, 1963), Austro-Hungarian-born American physicist and electronic engineer. Lilienfeld is credited with the first patents on the field-effect transistor (1925) and electrolytic capacitor (1931).

**Thomas Alva Edison** (February 11, 1847 – October 18, 1931): Light bulb, carbon microphone, DC power plant and distribution, sound recording, motion pictures, fluoroscope, etc.

**Nikola Tesla** (10 July 1856 – 7 January 1943) Serbian American inventor, electrical engineer, mechanical engineer, physicist, and futurist best known for his contributions to the design of the modern alternating current (AC) electricity supply system. Battle of AC vs. DC with Edison. The tesla is the derived SI unit of magnetic field.

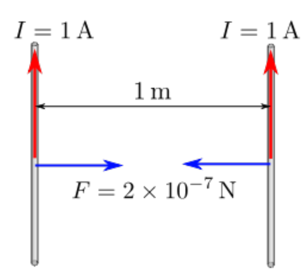
## Voltage

"Voltage is a measure of electric potential, … a type of potential energy, and refers to the energy that could be released if electric current is allowed to flow… One volt is defined as the difference in electric potential between two points of a conducting wire when an electric current of one ampere dissipates one watt of power between those points. It is also equal to the potential difference between two parallel, infinite planes spaced 1 meter apart that create an electric field of 1 newton per coulomb. Additionally, it is the potential difference between two points that will impart one joule of energy per coulomb of charge that passes [between the two points]. Voltage can be expressed in terms of SI base units (m, kg, s, and amperes A) as

$$\mathrm{V} = \frac{\text{Potential Energy}}{\text{charge}} = \frac{\mathrm{N \cdot m}}{\text{coulomb}} = \frac{\mathrm{kg \cdot m \cdot m}}{\mathrm{s^2 \cdot A \cdot s}} = \frac{\mathrm{kg \cdot m^2}}{\mathrm{A \cdot s^3}}$$

"Voltage for alternating current almost never refers to the voltage at a particular instant, but instead is the root mean square (RMS) voltage… In most cases, the fact that a voltage is an RMS voltage is not explicitly stated, but assumed.[2]

Voltages vary from a few mV in nerve conduction and ECGs to > 1 GV in lightning arising from positively charged cloud tops.[3]




## Current and Charge

Ampère's force law states that there is an attractive or repulsive force between two parallel wires which each carry an electric current. This force is used in the formal definition of the ampere."[4] The ampere is "the basic unit of electrical current in the

[2] Quoted and paraphrased from https://en.wikipedia.org/wiki/Volt including diagram
[3] https://en.wikipedia.org/wiki/Lightning#Positive_and_negative_lightning and http://www.srh.noaa.gov/jetstream//lightning/positive.htm
[4] https://en.wikipedia.org/wiki/Ampere incl. diagram and paraphrased text

International System of Units (SI), equivalent to one coulomb per second, formally defined to be the constant current [i.e., equal in amount and direction] which if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed one meter apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$newton per meter of length."[5]

The SI unit of charge, the coulomb, is "equal to the quantity of charge transferred in one second across a conductor in which there is a constant current of one ampere."[6] In general, charge Q is determined by steady current I flowing for a time t, specifically $Q = I \cdot t$.
One coulomb (of positive charge) ≈ $6.241509 \bullet 10^{18} \bullet$(charge of proton)
One negative coulomb (i.e., of negative charge) ≈ $6.241509 \bullet 10^{18} \bullet$(charge of electron).
It may also be said that one coulomb is ≈ the magnitude (absolute value) of electrical charge in $6.241509 \bullet 10^{18}$ protons or electrons.[7]

## Relating Voltage, Current, and Power[8]

For DC power:

$$P = V\,I$$

where

P = power consumed by a load (in watts, where 1 W = 1 joule/sec = 1 kg $m^2$ $s^{-3}$).
W (often after a number) is the abbreviation for watts of power
V = Voltage across a load (volts, where 1 V = 1 $kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$).
V in upper case is used (often after a number) as the abbreviation for volts
I = Current (amperes).
A (often after a number) is the abbreviation for amperes of current

In the textbook *EEAI3*, P V I represent rms magnitude values for AC and p v i represent instantaneous values. For AC Power, the formula applies for instantaneous p, v, and i (with complications to follow).

## Resistance and Conductance, Resistance of a Wire (Pouillet's law)

Resistance is expressed in ohms and is in many cases approximately constant within a certain range of voltages, temperatures, and other parameters. The units of resistance may be expressed as[9]

$$\Omega = \frac{V}{A} = \frac{1}{S} = \frac{W}{A^2} = \frac{V^2}{W} = \frac{s}{F} = \frac{J \cdot s}{C^2} = \frac{kg \cdot m^2}{s \cdot C^2} = \frac{J}{s \cdot A^2} = \frac{kg \cdot m^2}{s^3 \cdot A^2}$$

where A = amperes, C = coulombs, F = farads, J = joules, s = seconds, S = Siemens, V = volts, W = watts

Resistance of a wire (or other uniform homogeneous conductors with uniform cross section) is given by Pouillet's law:

$$R_{wire} = \rho \, \frac{l}{A}$$

where

$R_{wire}$ = Resistance of wire or conductor (Ω ohms)
ρ = Resistivity of the wire or conductor material (Ω-m)
l = Length of wire or conductor (m)
A = Wire or conductor uniform cross sectional area ($m^2$)

5 http://dictionary.reference.com/browse/ampere
6 http://dictionary.reference.com/browse/coulomb?s=t
7 https://en.wikipedia.org/wiki/Coulomb
8 Course lecture 1.pptx
9 https://en.wikipedia.org/wiki/Ohm including diagram and some paraphrases text.

A cube with faces of 1m and with resistance of 1Ω across opposite face sheet contacts has resistivity ρ = 1 ohm-m.

Electrical conductivity σ = 1/ρ (expressed in SI as siemens / m (S/m) where S = $\Omega^{-1}$ = I/V

Selected resistivities at 20 °C:[10]

| Substance | Resistivity ρ (ohm-m) |
|---|---|
| C (Graphene) | $1.00\times10^{-8}$ |
| Cu Copper | $1.68\times10^{-8}$ |
| Ag Silver | $1.49\times10^{-8}$ |
| Au Gold | $2.44\times10^{-8}$ |
| Al Aluminum | $2.82\times10^{-8}$ |
| W Tungsten | $5.60\times10^{-8}$ |
| Steel, Carbon (1010) | $1.43\times10^{-7}$ |
| Steel, Stainless<br>18% Cr/ 8% Ni austenitic | $6.90\times10^{-7}$ |
| Carbon, Amorphous | $5.00\times10^{-4}$ to $8.00\times10^{-4}$ |
| Water, Sea | $2.00\times10^{-1}$ |
| Water, Drinking | $2.00\times10^{1}$ to $2.00\times10^{3}$ |
| Air | $1.30\times10^{16}$ to $3.30\times10^{16}$ |

## Ohm's Law (relating V, I, and R) and Power P

Ohm's law (and its variations) relates V, I, and R. We add here the definition of Power P = V I, and the resulting relationships among P, V, I, and R.[11]

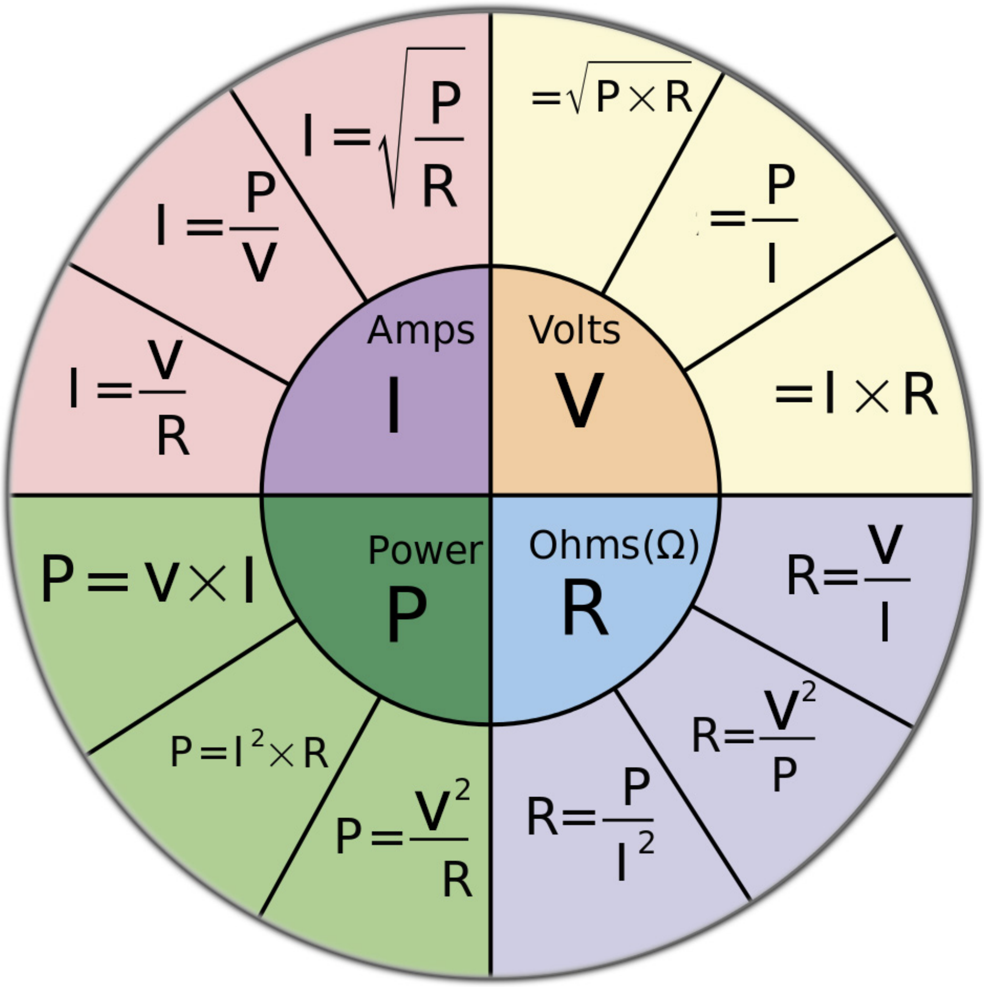
Current is proportional to V and inversely proportional to R, specifically I = V/R

where

I = Current (amperes)
V = Voltage (volts)
R = Resistance (ohms)

In the following VIRP wheel, power consumed by a static resistance is also shown.

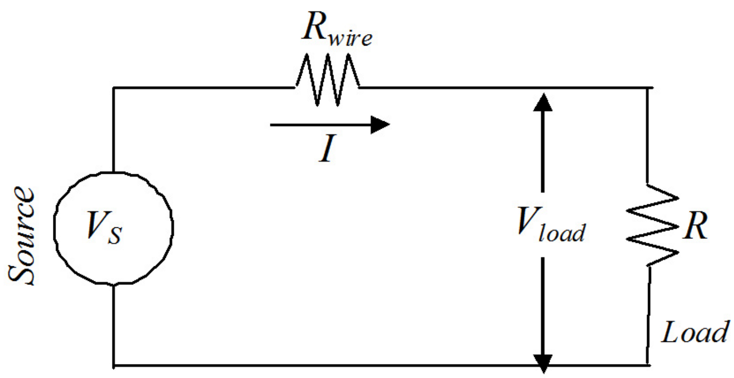
[10] https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity
[11] https://commons.wikimedia.org/wiki/File:Ohm's_Law_Pie_chart.svg , diagram slightly modified MCM, instantaneous quantities




## Voltage Divider Circuit Showing Voltage Drop from Line Resistance[12]

In the following circuit having only static resistances, the current I passes thru the wire resistance and the load resistance.

[12] ibid.




$$V_{load} = V_S - I\ R_{wire}$$

$$V_{load} = I\ R = \frac{V_S}{R + R_{wire}}\ R$$

Thus, the greater the wire resistance, the lower the current and the voltage attained across the load.

# Alternating Current Phases and Analysis

Material in this AC section derives in part from chapter 1, chapter 2, and chapter 7.

## AC Nomenclature and Symbols

Nomenclature and symbols used by the textbook[13]

| | | | |
|---|---|---|---|
| Instantaneous current | $i$ | Instantaneous voltage | $v$ |
| Average current | $I_{ave}$ | Average voltage | $V_{ave}$ |
| Maximum (peak) current | $I_{max}$ | Maximum (peak) voltage | $V_{max}$ |
| Current magnitude in root mean square | $I$ | Voltage magnitude in root mean square | $V$ |
| Phasor current (complex form) | $\bar{I}$ | Phasor voltage (complex form) | $\bar{V}$ |
| Complex impedance | $\bar{Z}$ | Complex power | $\bar{S}$ |
| Magnitude of impedance | $Z$ | Magnitude of complex power | $S$ |

Note that instantaneous quantities i and v are expressed in lower case, non-RMS averages and max values are spelled out, RMS magnitudes of I and V and magnitude of Z are shown as unadorned upper case, and Phasors $\bar{I}$ and $\bar{V}$ and Complex quantities $\bar{S}$ and $\bar{Z}$ are written with a bar over the upper case letter (and have rms values in magnitude). V in upper case is also used (after a number) as the abbreviation for volts, A (after a number) is the abbreviation for amperes of current, and W (after a number) is the abbreviation for watts of power. Average voltage is typically 0 for symmetrical sine wave AC.

[13] *EEAI3* p. 213

## Alternating Current Waveform Equation

$v = V_{max} \sin \omega t$

where

| | | |
|---|---|---|
| $v$ = instantaneous voltage | | (Volts) |
| $\omega$ = angular frequency where $\omega = 2\pi f = 2\pi/T$ | | (radians/s) |
| $f$ = waveform frequency = $\omega/2\pi = 1/T$ | | (Hz or $s^{-1}$) |
| $T$ = waveform period = $1/f = 2\pi/\omega$ | | (s) |

For most US power calculations, $\omega = 2\pi f = 2\pi*60 \approx 376.991$ radians/s ≈ 377 radians/s. Note that $\omega t$ has units of radians (or is sometimes expressed in degrees)

## RMS AC Voltage V

This is given by the sqrt of 1/T times the integral of the square of the instantaneous voltage v over a complete period of duration T. The RMS may be calculated for any periodic waveform that is defined mathematically. The derivation for periodic sinusoidal waveforms may be given as follows:[14]

[14] http://www.raeng.org.uk/publications/other/8-rms

So the RMS value $y_{RMS}$ is given by:

$$y_{RMS} = \sqrt{\frac{1}{T}\int_0^T y^2(t)dt} = \sqrt{\frac{1}{T}\int_0^T A^2 \sin^2 \omega\, t dt} \quad \dots (3)$$

We know that $\cos 2\theta = 1 - 2\sin^2\theta$ , hence:

$$\sin^2 \omega\, t = \frac{1}{2}(1 - \cos 2\omega\, t) \dots (4)$$

Substituting (4) into (3), we get, in sequence

$$y_{RMS} = \sqrt{\frac{1}{T}\int_0^T \frac{A^2}{2}(1 - \cos 2\omega\, t)dt}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T}\int_0^T (1 - \cos 2\omega\, t)\ dt}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T}\left[t - \frac{\sin 2\omega\, t}{2\omega}\right]_0^T}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2T}\left[T - \frac{\sin 2\omega\, T}{2\omega}\right]}$$

$$y_{RMS} = \sqrt{\frac{A^2}{2} - \frac{A^2 \sin(2\omega\, T)}{4\omega\, T}} \quad \dots (5)$$

As $T \to \infty$ , the second (oscillatory) term in equation (5) tends to zero. Hence, we obtain:

$$y_{RMS} = \frac{A}{\sqrt{2}}$$

$$\Rightarrow \text{RMS Value of y} = \frac{1}{\sqrt{2}} \times \text{Peak Value of } y$$

Note: In this derivation taken from the web, if one instead simply integrated over exactly one period T, the integral of the cos 2ωt term would be over exactly 2 periods and thus would be zero, yielding the same final result.
All AC voltages in US household and industrial power circuits and equipment, unless otherwise stated, are expressed in rms values. Thus, nominal 120 V is 120 V rms, so $V_{max} = 120\sqrt{2} = 169.7$ volts.[15] Voltages at wall socket active single plugs varies from +170 to -170 V relative to ground potential.

$V_{max}$ values tend to vary somewhat due to transients, varying loads, and harmonics, more so than V (i.e., $V_{rms}$). It follows that $V_{rms} = V_{max} / \sqrt{2}$.

US AC frequencies are 60 Hz, Europe and other countries are often 50 Hz.

[15] *EEAI3* p. 214-215

## Phasors, Complex Impedance, and Phase Shifts from Inductors and Capacitors

### Phasors

A phasor [from "phase vector"] is a graphical representation which depicts the magnitude and phase shift of an AC waveform while hiding the instantaneous location within the cycle represented by the sine or cosine ωt term. They are represented in angle notation by $\overline{A} = A\angle\theta$, where A is the magnitude and θ is the angle with respect to the reference (positive for leading, negative for lagging). These may be converted to complex numbers to analyze how phasors for R, C, and L add, subtract, multiply, etc.

Phasor complex arithmetic is done as follows (where $\theta_1$ is the phase angle for A, etc.):

$$\overline{A} = A\angle(\theta_1) = A[\cos\theta_1 + j\sin\theta_1] = X + jY$$

where $j = \sqrt{-1}$, and X and Y are real and imaginary components, resp.
(the traditional math symbol $i=\sqrt{-1}$ is not used in EE to avoid confusion with current)

Multiplication: $\overline{A}\,\overline{B} = AB\angle(\theta_1 + \theta_2)$

Division: $\frac{\overline{A}}{\overline{B}} = \frac{A\angle\theta_1}{B\angle\theta_2} = \frac{A}{B}\angle(\theta_1 - \theta_2)$

Addition: $\bar{A} + \overline{B} = A[\cos\theta_1 + j\sin\theta_1] + B[\cos\theta_2 + j\sin\theta_2]$
$= (A\cos\theta_1 + B\cos\theta_2) + j\,(A\sin\theta_1 + B\sin\theta_2)]$

Subtraction $\bar{A} - \overline{B} = A[\cos\theta_1 + j\sin\theta_1] - B[\cos\theta_2 + j\sin\theta_2]$
$= (A\cos\theta_1 - B\cos\theta_2) + j\,(A\sin\theta_1 - B\sin\theta_2)]$

Complex conjugate: if $A = X + jY$: $A^* = X - jY$

Inverting a phasor: $\frac{1}{\overline{A}} = \frac{X}{X^2+Y^2} - j\frac{Y}{X^2+Y^2}$

A **phasor diagram** according to our textbook shows the voltage as a reference along the traditional horizontal x-axis, with length proportional to rms value. (Judging by the diagrams below, placing voltage on the x-axis is not a universal convention.) By convention, the direction of rotation in time is counterclockwise, so a lag such as a current lag is shown as a current arrow rotated in the clockwise direction relative to the reference quantity (here voltage), and a current lead is shown as a counterclockwise current arrow rotation.[16] For either case, the range of lag or leading angles is by convention $0 \le i \le 180°$. (A lag of more than 180 degrees would more likely be described as a lead of less than 180 degrees).

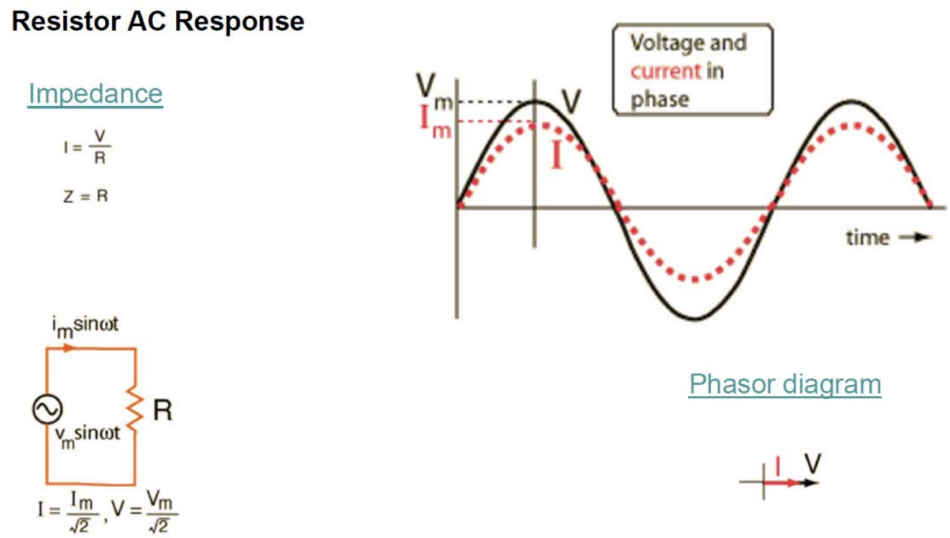
### Resistors

For a pure resistive (Ohmic) element with resistance R (ohms Ω), the instantaneous voltage across and current through the resistor (v and $i_R$) are:

$$v = V_{max}\sin\omega t = i_R R$$
$$i_R = {}^{V_{max}}\!/_{R}\,\sin\omega t$$

Phase and Phasor diagram are shown below (where $I_m = I_{max}$).[17] This resistance does not cause a phase shift and instantaneous $i_R$ is in phase with v. The phasor shows I and V pointing in the same direction. (These diagrams from the web use V and I for instantaneous values.)

[16] *EEAI3* p. 218 etc.
[17] https://www.eiseverywhere.com/file_uploads/dbb257afe85a9168908ca87f9c8e76d5_PhasorDiagrams.pdf




## Inductors

"The henry (symbol H) [plural henries per NIST] is the unit of electrical inductance in the International System of Units. The unit is named after Joseph Henry (1797–1878), the American scientist who discovered electromagnetic induction… The units may be expressed as

$$\mathrm{H} = \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{s}^2\cdot\mathrm{A}^2} = \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{C}^2} = \frac{\mathrm{J}}{\mathrm{A}^2} = \frac{\mathrm{T}\cdot\mathrm{m}^2}{\mathrm{A}} = \frac{\mathrm{Wb}}{\mathrm{A}} = \frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}} = \frac{\mathrm{s}^2}{\mathrm{F}} = \frac{1}{\mathrm{F}\cdot\mathrm{Hz}^2} = \Omega\cdot\mathrm{s}$$

where A = amperes, C = coulombs, F = farads, H = Henries, J = joules, s = seconds, S = Siemens, T = teslas (magnetic flux density or magnetic field strength), V = volts, W = watts, Wb = webers (magnetic flux).[18]

The magnetic permeability [$\mu_0$] of a classical vacuum is defined as exactly $4\pi\times10^{-7}$ N/A$^2$ or H m$^{-1}$ (henry per metre).[19]

Inductance is often symbolized as L. For a pure inductive element or load (with no resistance or capacitance) having inductance L (in henries H), the instantaneous v across the inductor and instantaneous i through it are:

$$v = V_{max}\sin\omega t \;=\; L\frac{dI_L}{dt}$$
$$i_L = -{}^{V_{max}}/_{\omega L}\,\cos\omega t = -{}^{V_{max}}/_{X_L}\,\cos\omega t$$

The quantity $X_L \equiv \omega t$ is the magnitude of the **inductive reactance** of the inductor.

This inductor causes a phase shift and $i_L$ is not in phase with v. Rather, $i_L$ lags the voltage by 90º (or equivalently v leads $i_L$ by 90º).

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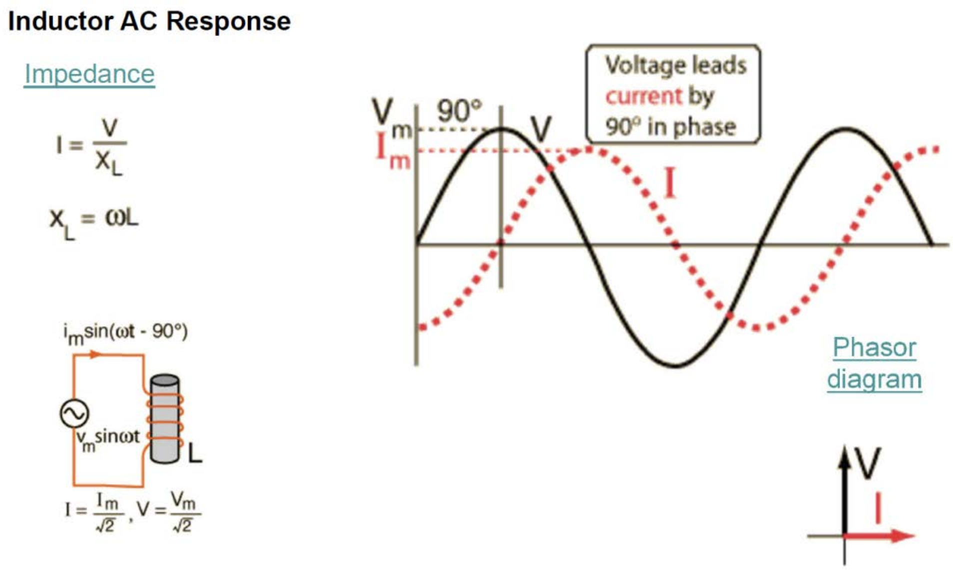
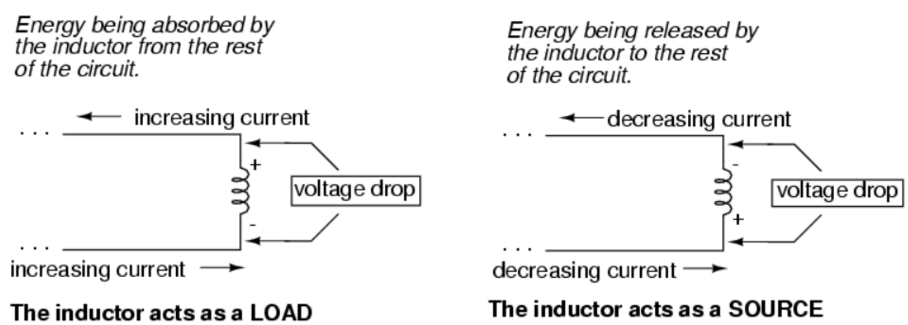
18 https://en.wikipedia.org/wiki/Henry_%28unit%29 including diagram and paraphrased text.
19 https://en.wikipedia.org/wiki/Vacuum_permeability




The phasor diagram above shows I lagging V by 90 degrees (again, lag is in the counterclockwise direction).

The inductor resists the buildup of current in response to an applied voltage, thus a time delay exists before current reaches a maximum. "According to Lenz's law the direction of induced e.m.f [electromotive force] is always such that it opposes the change in current that created [the e.m.f.] As a result, inductors always oppose a change in current, in the same way that a flywheel oppose a change in rotational velocity."[20] Inductive reactance is an opposition to the change of current through an element.

> "Because inductors store the kinetic energy of moving electrons in the form of a magnetic field, they behave quite differently than resistors (which simply dissipate energy in the form of heat) in a circuit. Energy storage in an inductor is a function of the amount of current through it... Inductors react against changes in current by dropping voltage in the polarity necessary to oppose the change. When an inductor is faced with an increasing current, it acts as a load: dropping voltage as it absorbs energy (negative on the current entry side and positive on the current exit side, like a resistor). When an inductor is faced with a decreasing current, it acts as a source: creating voltage as it releases stored energy (positive on the current entry side and negative on the current exit side, like a battery). The ability of an inductor to store energy in the form of a magnetic field (and consequently to oppose changes in current) is called inductance."[21]




## Capacitors

One farad is defined as the capacitance of a capacitor across which, when charged with one coulomb of electricity, there is a potential difference of one volt. Conversely, it is the capacitance which, when charged to a potential difference of one volt, carries a charge of one coulomb. The units of capacitance are given by

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[20] https://en.wikipedia.org/wiki/Inductor

[21] http://www.allaboutcircuits.com/textbook/direct-current/chpt-15/magnetic-fields-and-inductance/ diagrams slightly modified by MCM

$$F = \frac{A \cdot s}{V} = \frac{J}{V^2} = \frac{W \cdot s}{V^2} = \frac{C}{V} = \frac{C^2}{J} = \frac{C^2}{N \cdot m} = \frac{s^2 \cdot C^2}{m^2 \cdot kg} = \frac{s^4 \cdot A^2}{m^2 \cdot kg} = \frac{s}{\Omega} = \frac{s^2}{H}$$

where A = amperes, C = coulombs, F = farads, H = Henries, J = joules, s = seconds, S = Siemens, V = volts, W = watts.[22]

Capacitance is often symbolized as C.

For a purely capacitive element or load (with no resistance or inductance) having capacitance C (in farads F), the instantaneous v across the capacitor and i into it are:

$$v = V_{max} \sin \omega t \; = \frac{1}{C} \int i_C dt$$

Integrating,

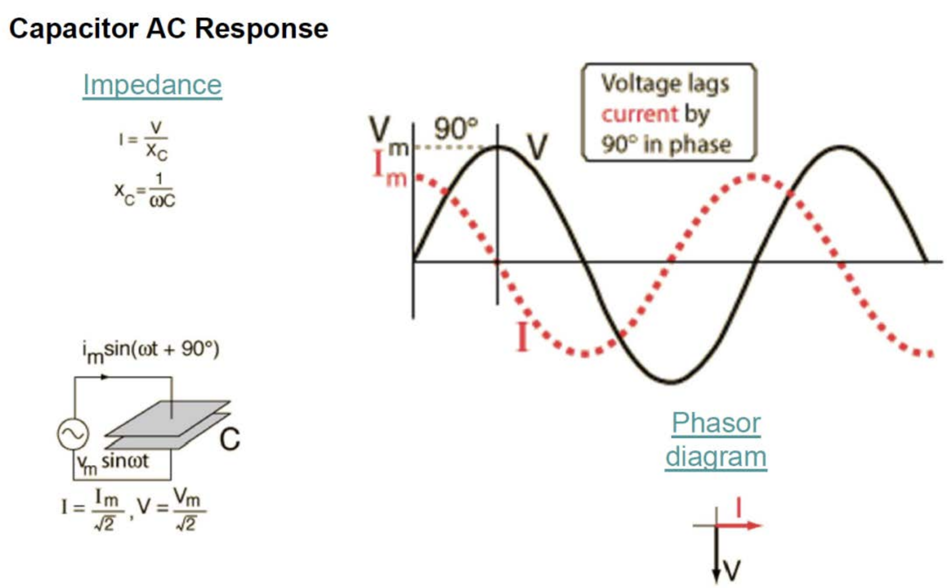
$$i = C \frac{dv}{dt}$$

$$i_C = \omega \, C \, V_{max} \cos \omega t = {}^{V_{max}}/_{X_C} \; \cos \omega t$$

The quantity $X_C \equiv 1/\omega t$ is the magnitude of the **capacitive reactance** of the capacitor. Capacitive reactance is an opposition to the change of voltage across an element.

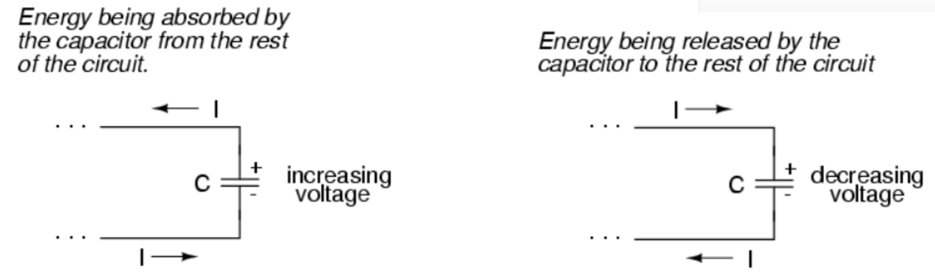
This capacitor causes a phase shift and $i_C$ is not in phase with v. Rather, $i_C$ leads the voltage by 90º (or equivalently v lags $i_C$ by 90º). The current must flow in before voltage is built up across the capacitor plates.

The phasor diagram shows I lagging V by 90 degrees (again, lag is in the counterclockwise direction).




"When the voltage across a capacitor is increased, it draws current from the rest of the circuit, acting as a power load. In this condition the capacitor is said to be charging, because there is an increasing amount of energy being stored in its electric field. Note the direction of electron current with regard to the voltage polarity [in the diagram to follow]. Conversely, when the voltage across a capacitor is decreased, the capacitor supplies current to the rest of the circuit, acting as a power source. In this condition the capacitor is said to be discharging. Its store of energy—held in the electric field—is decreasing now as energy is released to the rest of the circuit. Note the direction of electron current with regard to the voltage polarity."[23]

[22] https://en.wikipedia.org/wiki/Farad including diagram and paraphrased text
[23] http://www.allaboutcircuits.com/textbook/direct-current/chpt-13/electric-fields-capacitance/ diagrams slightly modified by MCM




**The capacitor acts as a LOAD** **The capacitor acts as a SOURCE**

## Complex Impedance

**Total Reactance** $\overline{X} = \overline{X_L} + \overline{X_C} = X_L\angle + 90° + X_C\angle - 90° = j(X_L - X_C)$ in phasor & complex notation, resp.

The magnitude of Total Reactance X = ωL - 1/ωC.

where $X_L \equiv \omega t$ is the magnitude of the inductive reactance $\overline{X_L}$ , and

$X_C \equiv 1/\omega t$ is the magnitude of the capacitive reactance $\overline{X_C}$

These are all expressed as ohms. Magnitudes $X_L$ and $X_C$ are both positive scalar quantities by convention, but the minus sign in the right part of the formula for total magnitude arises from the negative (lagging) phasor angle for $X_C\angle - 90°$. When magnitude X is positive, the total reactance is said to be inductive; when X is negative, the total reactance is said to be capacitive.

**Total impedance** in ohms Ω for elements arranged in **series** (computed by addition of phasors) is

$$\overline{Z} = R + \overline{X}_L + \overline{X}_C$$

$$\overline{Z} = R\angle 0° + X_L\angle + 90° + X_C\angle - 90° = R + j(X_L - X_C)$$ in phasor and complex notation, resp.

where $X_L \equiv \omega t$ is the magnitude of the inductive reactance $\overline{X_L}$

$X_C \equiv 1/\omega t$ is the magnitude of the capacitive reactance $\overline{X_C}$.

Total impedance for elements arranged in **parallel** (computed by addition of inverted phasors) is

$$\frac{1}{\overline{Z}} = \frac{1}{R} + \frac{1}{\overline{X}_L} + \frac{1}{\overline{X}_C}$$

**Resonant Frequency**: by adjusting ω until $X_L = X_C$, the total impedance is equal to the load resistance alone, and the resulting frequency $f_0$ is called the resonant frequency:[24]

$$f_0 = {}^{1}/_{2\pi LC}$$

Alternatively, the following quantities may be defined and used:[25]

**Conductance** $G = \frac{1}{R}$ (mhos = siemens. Note that G and R are real numbers)

**Inductive Susceptance** $\overline{B_L} = \frac{1}{\overline{X}_L}$ (mhos = siemens)

**Capacitive Susceptance** $\overline{B_C} = \frac{1}{\overline{X}_C}$ (mhos = siemens)

**Total Admittance** $\overline{Y} = G + \overline{B_L} + \overline{B_C} = G + j(B_C - B_L)$ (mhos = siemens)

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[24] *EEAI3* p. 225

[25] *EEAI3* p. 226

## Power

For sinusoidal waveforms, instantaneous power $\rho = vi = VI\,[\cos(\theta) - \cos(2\omega t - \theta)]$
where $\theta$ is the impedance phase angle.

For a purely resistive load, $\theta=0$ and $\rho = vi = VI\,[1 - \cos(2\omega t - \theta)]$, which is always positive

For a purely inductive load, $\theta=90$ degrees. $\rho = -VI\,[\sin(2\omega t)]$, which oscillates symmetrically between positive and negative values. The inductor consumes power as the voltage rises in the first 1/4 of the cycle, and returns power back in the 2nd 1/4 of the cycle, etc. On the average, the pure inductor does not consume any energy—it is "wattless".

For a purely capacitive load, $\theta=-90$ degrees. $\rho = VI\,[\sin(2\omega t)]$, which oscillates symmetrically between positive and negative values. The capacitor returns power as the voltage rises in the first 1/4 of the cycle, and returns power back in the 2nd 1/4 of the cycle, etc. On the average, the pure capacitor does not consume any energy. For loads that combine various amounts of $R + \overline{X}_L + \overline{X}_C$, the phase angle is not 0 and the average sum of instantaneous power is non-zero.[26]

The power that produces energy is the called **active power** or **real power**, expressed in watts W, and given by $P = VI\cos(\theta)$, where $\theta$ = overall phase angle. When $\theta=0$, P is simply VI watts (where V and I are both rms, approximately $0.707 \cdot V_{max}$ and $0.707 \cdot I_{max}$, respectively).

In contrast, the power that consumes or produces no net energy over multiple cycles is called **reactive power** or **imaginary power**. It is defined as $Q \equiv V \cdot I \sin\theta$, and expressed in Voltampere reactive VAr, kilovoltampere reactive kVAr, etc. $\theta$ = overall phase angle.

For an inductive load plus a resistance, inductive reactive power $Q_L = I^2 X_L$, the current lags the voltage by $\theta$, and the inductive reactive power leads the real power by 90°.

For a capacitive load plus a resistance, capacitive reactive power $Q_C = I^2 X_C$, the current leads the voltage by $\theta$, and the capacitive reactive power lags the real power by 90°.

**Complex power** phasor (aka apparent power) $\overline{S} \equiv \overline{V} \cdot \overline{I}^* = P + jQ = P + j(Q_L - Q_C)$, expressed as voltampere VA, kVA, etc. With this notation, the magnitudes of reactive power $Q_L$ of an inductor and $Q_C$ of a capacitor are both positive but because of the phase angles, that of the capacitor appears with a minus sign as discussed above.

**Power Factor pf** = $\frac{P}{S} = \frac{R}{Z} = \cos(\theta)$ may be lagging or leading depending on the angle of I wrt V: When I leads V, the pf is leading. Reactive power does not do work (thus it does not generate revenues to the utility), and its presence can cause problems:

It increased losses in the transmission line (due to increased current with $I^2R$ losses),
It reduces spare capacity of the line (due to increased current), and
It reduces the voltage across the load.

**Load Voltage**: The magnitude of load voltage is given by

$$V_{load} = V_S \Big/ \sqrt{\left(\frac{R_{wire}}{X_L}\right)^2 + \left(1 + \frac{X_{wire}}{X_L}\right)^2}$$

The load voltage falls when $X_L$ decreases (more reactive load is added). Only with infinite $X_L=\infty$ (no reactive load) is the load voltage equal to the source voltage.[27]

Power factor correction may be introduced into transmission and distribution lines and other circuits in the form of added parallel capacitance across the load, in order to reduce the power factor angle and offset the inductive reactance present.[28]

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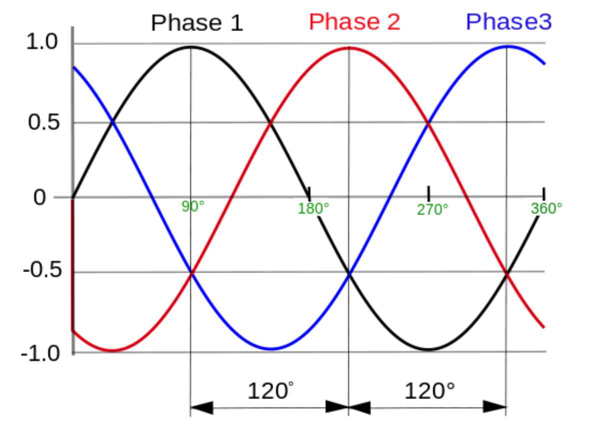
[26] *EEAI3* p. 228-230
[27] *EEAI3* p. 233-236
[28] *EEAI3* p. 238-240

**Energy Consumed**: This is simply $E = \int_0^{\tau} Pdt$ where P is instantaneous real power (i.e., adjusted for power factor) and τ is the time interval of interest. For discrete power levels, the sum rather than the integral applies.[29]

## Three-Phase Systems

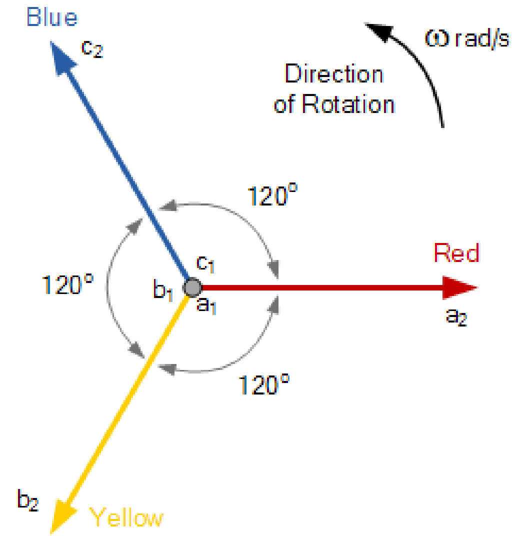
(This is a complex subject and I have provided only limited discussion, partly derived from chapter 8.) These systems are common in electrical generation, transmission, transformers, and industrial and commercial applications (including manufacturing, hospitals, and farming). 3-Phase current is generated in generators typically with 3 coils in the stator, each providing a *phase* of voltage as the magnetized rotor spins within. The 3 phases generated are "balanced" if the waveform is sinusoidal, the magnitudes of the rms voltages of the phases are equal, and the phases are separated by 120°.[30]




In 3-phase power generation, 3 phases of voltage are generated with phases 120° apart. In feeding a balanced and linear load, the sum of the instantaneous currents of the three conductors is zero. The current in each conductor is equal in magnitude to, but with the opposite sign of, the sum of the currents in the other two. The return path for the current in any phase conductor is the other two phase conductors.[31]

A Wye connected generator provides 3 lines plus a Neutral conductor. A Delta connected generator provides 3 lines and has no Neutral conductor.[32]

The magnitude of line to line (phase to phase) voltage $V_{LL}$ is $V_{L\text{-}N} \cdot \sqrt{3}$.[33]




The phase voltages are all equal in rms magnitude V but only differ in their phase angle. The three windings of the coils are connected together at points, $a_1$, $b_1$ and $c_1$ to produce a common neutral connection for the three individual phases. Then if the red phase is taken as the reference phase, each individual phase voltage can be defined with respect to the common neutral.[34]
In phasor notation relative to $a_2$ (and with positive angles in clockwise direction), the phases are

| | |
|---|---|
| Phase $a_2$ (Red or Phase 1, $v_{a2a1}$) | $V \angle 0°$ |
| Phase $b_2$ (Blue or Phase 2, $v_{b2b1}$) | $V \angle +120°$ |
| Phase $c_2$ (Yellow or Phase 3, $v_{c2c1}$) | $V \angle -120°$ |

In 3-phase power transmission, the 3 phases of the wye or delta source are connected to 3 (often bundled) conductors which transmit the power over distance, with the neutral of the generator [for Wye generators] connected to ground (Earth).[35]

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[29] *EEAI3* p. 242-4
[30] *EEAI3* p. 252
[31] https://en.wikipedia.org/wiki/Three-phase_electric_power including diagram
[32] *EEAI3* p. 250
[33] *EEAI3* p. 254
[34] http://diodetech.blogspot.com/2013/07/phasor-diagram.html Text paraphrased plus diagram
[35] *EEAI3* p. 252,
also http://electronics.stackexchange.com/questions/124817/why-are-there-only-3-wires-on-this-power-line

In some cases, the source of power might be a balanced wye yet the load is in a balanced delta configuration.[36]

## Advantages of 3-phase over single-phase

- "Three phase power **transmission** has become the standard for power distribution. Three phase power generation and distribution is advantageous over single phase power distribution [because they transmit 3 times the power as single-phase lines].
- Three phase power distribution requires **lesser amounts of copper or aluminium** for transferring the same amount of power as compared to single phase power
- The size of a three phase **motor is smaller** than that of a single phase motor of the same rating. [Motors of higher HP are available with 3-phase.]
- Three phase **motors are self-starting** as they can produce a rotating magnetic field [a very important advantage]. The single phase motor requires a special starting winding as it produces only a pulsating magnetic field. [3-phase motors do not spark on startup.]
- In single phase motors, the power transferred in motors is a function of the instantaneous current which is constantly varying. Hence, single phase motors are more prone to vibrations. In three phase motors, however, the power transferred is uniform throughout the cycle and hence **[motor] vibrations are greatly reduced**.
- The **ripple factor** of rectified DC produced from three phase power is less than the DC produced from single phase supply. [Thus, with 6 peaks per cycle rather than 2 peaks, 3-phase is a steadier source of power.]
- Three phase motors have **better power factor regulation**.
- Three phase generators are smaller in size than single phase generators as winding phase can be more efficiently used. [Equivalently, a 3-phase **generator generates more power** than a single-phase generator occupying the same volume.]"[37]
- Both 3-phase and single phase equipment can be powered from a 3-phase supply, but not the opposite.
- The total three-phase power supplied to a balanced three-phase circuit remains constant.
- 3-phase power is more reliable: when one phase is lost, the other two phases can still deliver some power.[38]

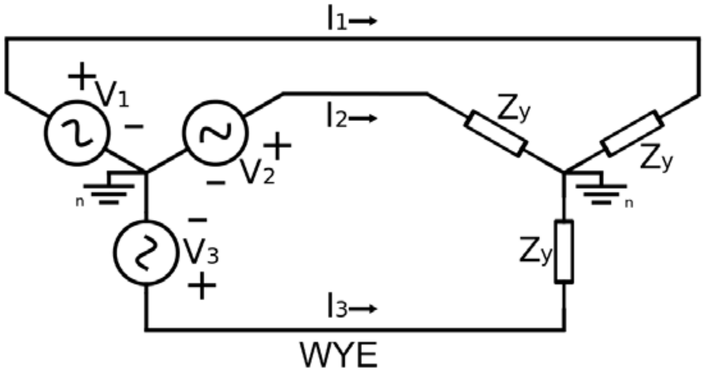
## Disadvantages of 3-phase over single-phase

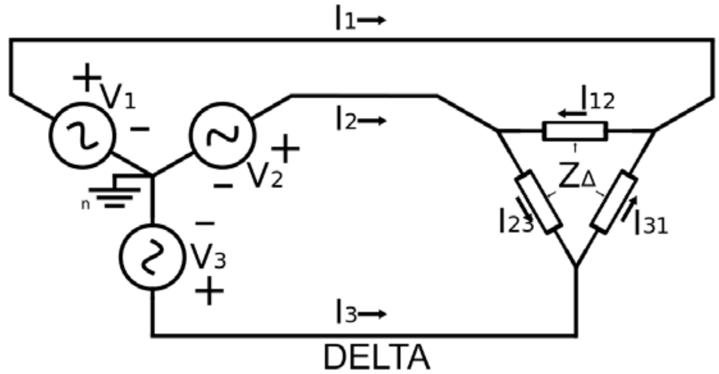
- Electrical supply, control, and end-devices are often more complex and expensive.[39] [However, costs for motors and for installation of equipment are often lower.][40] Three transformers are needed for voltage conversion for Wye 3-phase. Although only two are needed for delta 3-phase, but you cannot obtain as much power from a given size transformer as you can with the delta connection.[41]
- Failure of a 3-phase transformer is full failure. In contrast, when single single-phase transformers are used to convert 3-phase power, failure of one of the single-phase transformers leaves 2 single-phase transformers still operational.[42]

[36] *EEAI3* p. 63
[37] http://www.electrotechnik.net/2010/11/advantages-of-three-phase-power-over.html , also *EEAI3* p. 247
[38] *EEAI3* p. 247
[39] http://electrical-engineering-portal.com/single-phase-power-vs-three-phase-power
[40] https://www.linkedin.com/pulse/20141106222822-3267680-single-phase-vs-three-phase-power-what-you-need-to-know
[41] https://fairfld61.files.wordpress.com/2010/07/amta4-5-comparing-single-phase-and-three-phase-systems.ppt
[42] http://www.electricaltechnology.org/2012/02/advantages-of-three-phase-transformer.html

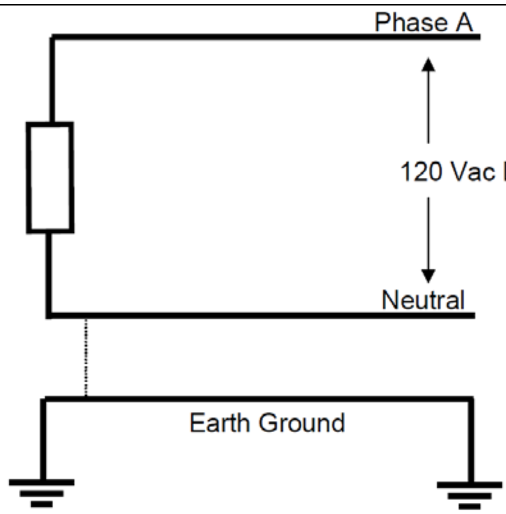
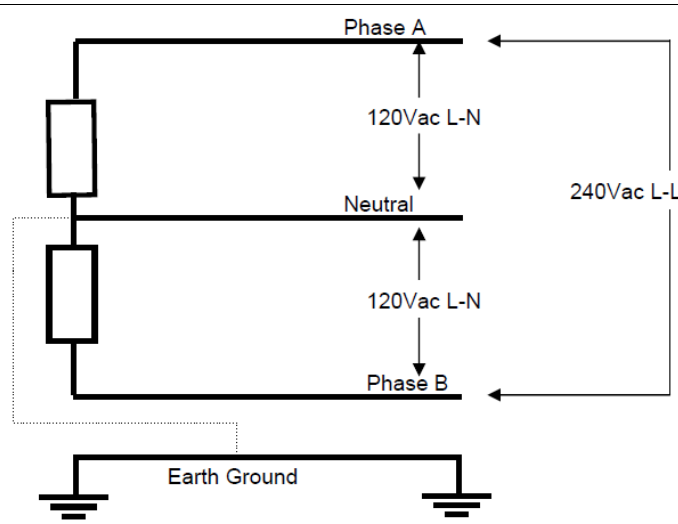
### 3-Phase Circuit Diagrams:

The following show in schematic form the generation and delivery to loads of 3-phase power in both Wye ("Y" or "star") and Delta load configurations:[43]

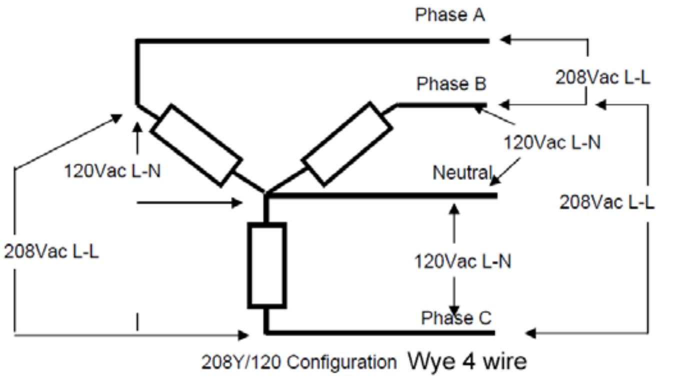


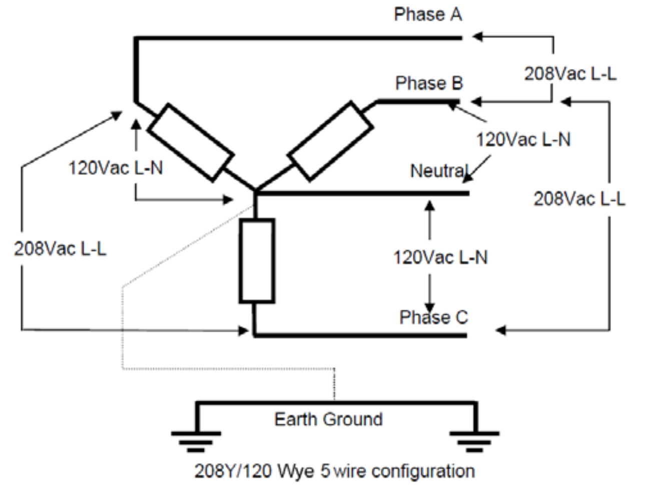

The following diagrams from another source show voltages across line to line and line to neutral for representative single phase and 3-phase Wye load (4- and 5-wire) and Delta load configurations:[44]

| | |
|---|---|
|  <br>Single Phase 120<br>Single phase 120V "house current" with safety enhancing neutral to Earth Ground connection (dotted line). 120 volt AC Voltage "Vac L-N" is "Line to Neutral" (aka "Phase to neutral", here Phase A to Neutral). "Neutral is a circuit conductor that normally carries current, and is connected to ground (earth) at the main electrical panel."[45] |  <br>Single Phase 120/240<br>Single phase 120/240V "house current" or "split phase". This configuration has 2 voltage hot "lines" (Phase A and Phase B here) that have phase 180° apart. Safety enhancing neutral to ground connection (dotted line) also shown. 120V is available from Phase A or B to Neutral and 240 volt AC Voltage "Vac L-L" is "Line to Line" (aka "Phase to Phase voltage", here Phase A to Phase B). |

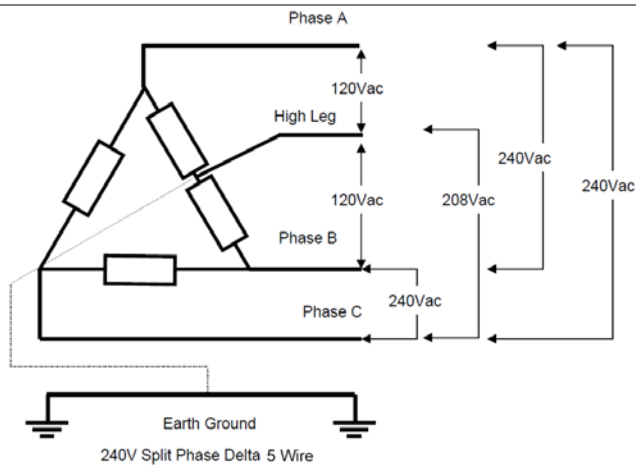
[43] https://en.wikipedia.org/wiki/Three-phase_electric_power
[44] Ametek Programmable Power, www.programmablepower.com/support/FAQs/DF_AC_Distribution.pdf , all images slightly modified MCM, text paraphrased
[45] https://en.wikipedia.org/wiki/Ground_and_neutral



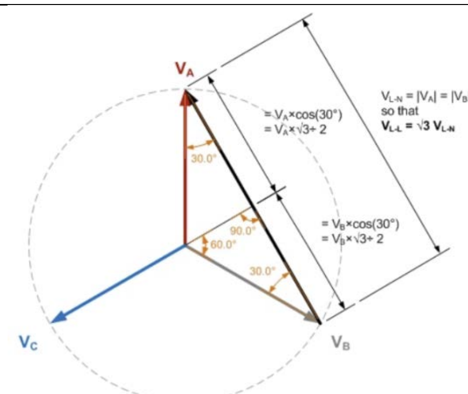

3-phase 4-wire Wye (i.e., having Y-shaped loads and phases): This load configuration has 3 "current carriers" (aka "lines" or "phases") which are 120° apart in phase. The fourth conductor is the "neutral" wire, which carries little or no current if the 3 phases are balanced (matched) in load. This 208Y/120 (aka 120/208Vac or 208Y/120 Wye ) configuration is common in the US. The 208 L-L rms voltage value (voltage across any 2 "lines") derives from $V_{L-N} \cdot \sqrt{3} = 120\sqrt{3}$. There is no Earth Ground here.




Same but showing a 5th wire, the Earth Ground, which is usually connected to neutral at the main electrical (circuit breaker) panel. This is the usual Wye load configuration in the US.




The 240V Split Phase Delta (aka dog leg or stinger leg) is one of several possible Delta load configurations (named for the delta or triangular shape of the loads and phases). Delta configurations are less common than Wye. There is no Neutral. One load is center tapped to provide two phases with 120Vac and a High Leg which provides 208Vac in addition to 240 Vac.




Computation of balanced 3-phase line-to-line voltage $V_{L-L}$ in terms of phase voltage $V_{L-N}$:[46]

$$V_{L-L} = \sqrt{3} \times V_{L-N}$$

The line-to-line voltage—for example $V_{A-B}$ for transmission lines—leads the phase voltage $V_A$ by 30°. (Note the order of subscripts).

[46] http://electronics.stackexchange.com/questions/92678/three-phase-power-supply-what-is-line-to-line-voltage

### 3-phase Current

Wye: In Wye balanced loads, the impedances of the 3 loads are identical. (For residential loads, each load represents a group of houses such that the resultant impedances are approximately equal.) The load impedances $\bar{Z}$ are associated with a impedance (phase shift) angle φ that is the same for each load. For a given line or phase (a), the current with respect to the phase voltage is given by:

$$\bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}_{an}} = \frac{\bar{V}_{an}}{\bar{Z}} = \frac{\bar{V}_{ph}\angle\theta}{Z\angle\varphi} = \frac{V_{ph}}{Z}\angle(\theta - \varphi)$$

where $V_{ph}$ is the rms voltage of phase a. Thus, the phase current lags or leads the voltage by φ. The other phase currents are separated by 120° in phase. Line currents are equal to the corresponding currents of the loads. All 3 phase currents are equal in magnitude. By Kirchoff's current rule, the sum of the phasors of the phase currents is 0:

$$\bar{I}_n = \bar{I}_a + \bar{I}_b + \bar{I}_c = 0$$

Thus, for Wye systems, the neutral conductor carries 0 current. The neutrals at the source and loads can be connected to local earth grounds.[47]

**Delta**: For Delta connected loads, the line-to-line currents are given by:

$$\bar{I}_{ab} = \bar{I}_a + \bar{I}_{ca} \text{ etc.}$$

In balanced systems, the loads are equal and the line-to-line voltages are equal, so the load current are also equal.[48]

It is possible to have mixed circuits—such as a Delta source but Wye load or vice versa—or even more complexly mixed arrangements.

It is also possible to find a Wye load connection which is equivalent to a Delta load connection and can therefore be used to represent it for the purpose of making calculations. The opposite is also possible, i.e., finding a Delta configuration equivalent to a Wye configurations. This mathematical technique is called a Wye-Delta transformation (or Y-Δ transform), or more precisely either a Δ-load to Y-load transformation or a Y-load to Δ-load transformation.[49]

### 3-phase Power

The power consumed in a balanced 3-phase load is the sum of the powers in each load. For each phase,

Real power $P_{ph} = V_{ph} I_{ph} \cos\theta$

Reactive power $Q_{ph} = V_{ph} I_{ph} \sin\theta$

where θ is the power factor angle (angle between load voltage magnitude $V_{ph}$ and load current magnitude $I_{ph}$)

Total Wye 3-phase real power is $3 \cdot P_{ph}$., etc. For balanced loads, the power may be expressed as

Real power $P_{tot} = 3 \cdot P_{ph} = 3V_{ph} I_{ph} \cos\theta = 3\frac{V_{LL}}{\sqrt{3}} I_{ph} \cos\theta = \sqrt{3} V_{LL} I_L \cos\theta$

Reactive power $Q_{tot} = \sqrt{3} V_{LL} I_L \sin\theta$

where $V_{LL}$ is line-to-line (phase to phase) rms voltage magnitude and $I_L$ is line (phase) current.

Total Delta load 3-phase power is also given by

Real power $P_{tot} = \sqrt{3} V_{LL} I_L \cos\theta$

Reactive power $Q_{tot} = \sqrt{3} V_{LL} I_L \sin\theta$

Thus θ is the angle of the load impedance)[50]

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[47] *EEAI3* p. 258-9
[48] *EEAI3* p. 260
[49] *EEAI3* p. 265 and https://en.wikipedia.org/wiki/Y-%CE%94_transform
[50] *EEAI3* p. 269

# Energy Resources and Overall Energy Utilization

This is a limited summary derived in part from chapter 3, with some statistics updated from other sources.

The International Energy Agency (IEA) summary of energy statistics may be found here.[51]

## Primary Energy Sources

*Primary* energy sources are raw resources which are eventually transformed into *secondary* more convenient and/or usable *energy carriers* (such as electricity). Primary sources include *non-renewable primary sources* (fossil and mineral fuels such as uranium and thorium), along with *renewable primary sources* such as hydropower. "Primary energy is the energy embodied in natural resources prior to undergoing any human-made conversions or transformations. Examples of primary energy resources include coal, crude oil, sunlight, wind, running rivers [i.e., hydropower in the broad sense], vegetation, and uranium."[52] After conversion, the three major primary energy sources—fossil fuels, nuclear, and hydroelectric—contribute >99% of world electric energy generation.[53]

In somewhat greater detail, primary energy sources consist of

**Primary Fossil Fuels**: These are coal, crude oil, and natural gas

**Primary Nuclear Fuels**: The *fissile* isotopes (i.e., fissionable radioisotopes found in quantities in nature) are $^{238}U$ and $^{235}U$. The *fertile* (non-fissile) radioisotope of Thorium $^{232}Th$ is also a primary fuel found in nature. Plutonium isotopes do not occur naturally in significant amounts.[54]

Someday, primary *fusion* fuel, namely deuterium [$^{2}H$ or D] (when combined with synthetic tritium $^{3}H$), may become commercially viable for electrical energy-generation.[55]]

**Primary Renewable Resources**: These include solar photons, hydropower (in the broadest sense), wind; biomass; and geothermal reservoirs.

The following graph shows the relative contributions of the several primary energy sources :[56]

[51] IEA, International Energy Agency: Key World Energy Statistics 2014. http://www.iea.org/publications/freepublications/publication/KeyWorld2014.pdf

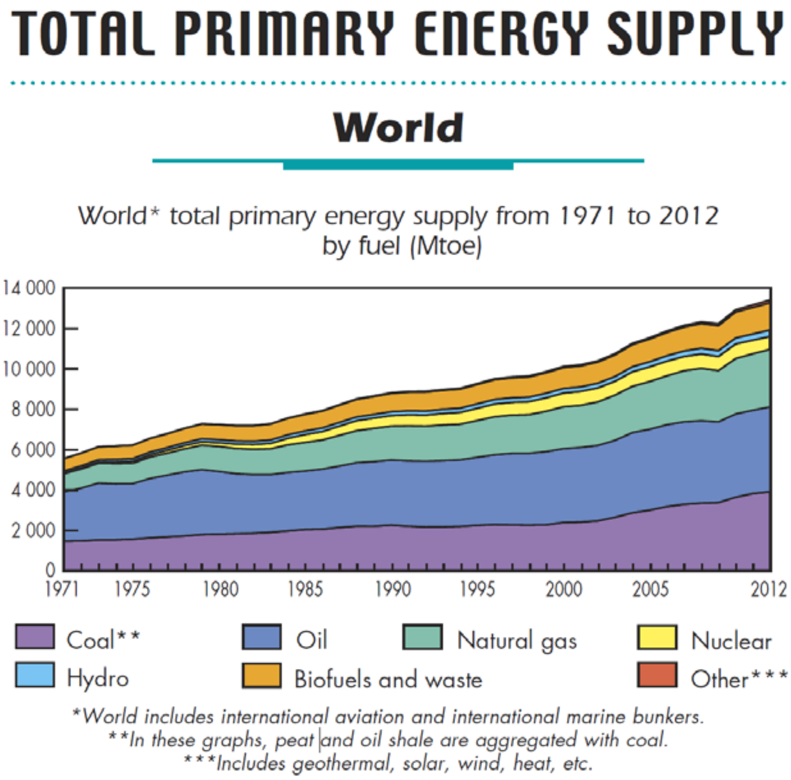
[52] http://www.eoearth.org/view/article/155350/

[53] *EEAI3* p. 41

[54] http://www.world-nuclear.org/info/current-and-future-generation/thorium/

[55] http://fusionforenergy.europa.eu/understandingfusion/merits.aspx , also https://www.iter.org/sci/fusionfuels

[56] http://www.iea.org/publications/freepublications/publication/KeyWorld2014.pdf , slightly modified MCM




Here, mtoe are millions of tonnes of oil equivalent (see below).

### Conversion of Primary Sources to Secondary Energy Carriers

[Thermodynamic Terminology has not been summarized here.] The Gibbs free energy G is the energy associated with a chemical reaction that can be used to do work. The free energy of a system is the sum of its enthalpy (H) plus the product of the temperature (Kelvin) and the entropy (S) of the system. The Gibbs free energy of the system is a state function because it is defined in terms of thermodynamic properties that are state functions. The change in the Gibbs free energy of the system that occurs during a reaction[57] is therefore equal to the change in the enthalpy of the system minus the change in the product of the temperature times the entropy of the system, or

$$\Delta G = \Delta H - \Delta(TS) \text{ or,}$$
$$\Delta G = \Delta H - T\Delta S \text{ (for constant T)}$$

Primary energy sources are converted to more readily usable secondary *carriers* of energy.

A major intermediate carrier is **thermal energy,** typically in the form of **steam** and/or **enthalpy**, which are used for providing **heating** and for turning **steam turbines** for **electrical power** generation**.**

**The secondary sources (carriers)** include the following:

**From Fossil Fuels**: Crude oil can be refined to **fuel oil** and other **refined fuels**, which ultimately is used to provide **thermal energy** or power **internal combustion engines**. Coal, oil, and natural gas are converted through burning to yield **thermal energy**, which can give rise to **mechanical work** or generation of **electricity**.[58]

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[57] http://chemed.chem.purdue.edu/genchem/topicreview/bp/ch21/gibbs.php
[58] https://en.wikipedia.org/wiki/Primary_energy

**From Nuclear Fuels**: **Thermonuclear fission** of primary fuel radioisotopes ($^{238}U$, $^{235}U$, and $^{232}Th$) and certain synthesized radioisotopes (especially $^{239}Pu$, $^{240}Pu$, and $^{238}Pu$, )[59] makes **thermal energy** which is used to generate **electricity**.[60]

**From Renewable Resources**:
• **Solar energy** provides
(1) **thermal energy** , some of which is used for generation of electricity
(2) **photovoltaic electricity (PV**);[61]
• **Hydropower** (including river flow, tidal excursion and wave action) generates **mechanical work** and/or **hydroelectric HE power**;[62]
• Wind generates **mechanical work** or **electricity**;[63]
• Biomass[64] (crop and forest residue, wood, ? charcoal, other waste, biogas [methane], celluosic ethanol[65]) generate **thermal energy** and/or **electricity**
• Geothermal generates **thermal energy** and/or **electricity**.

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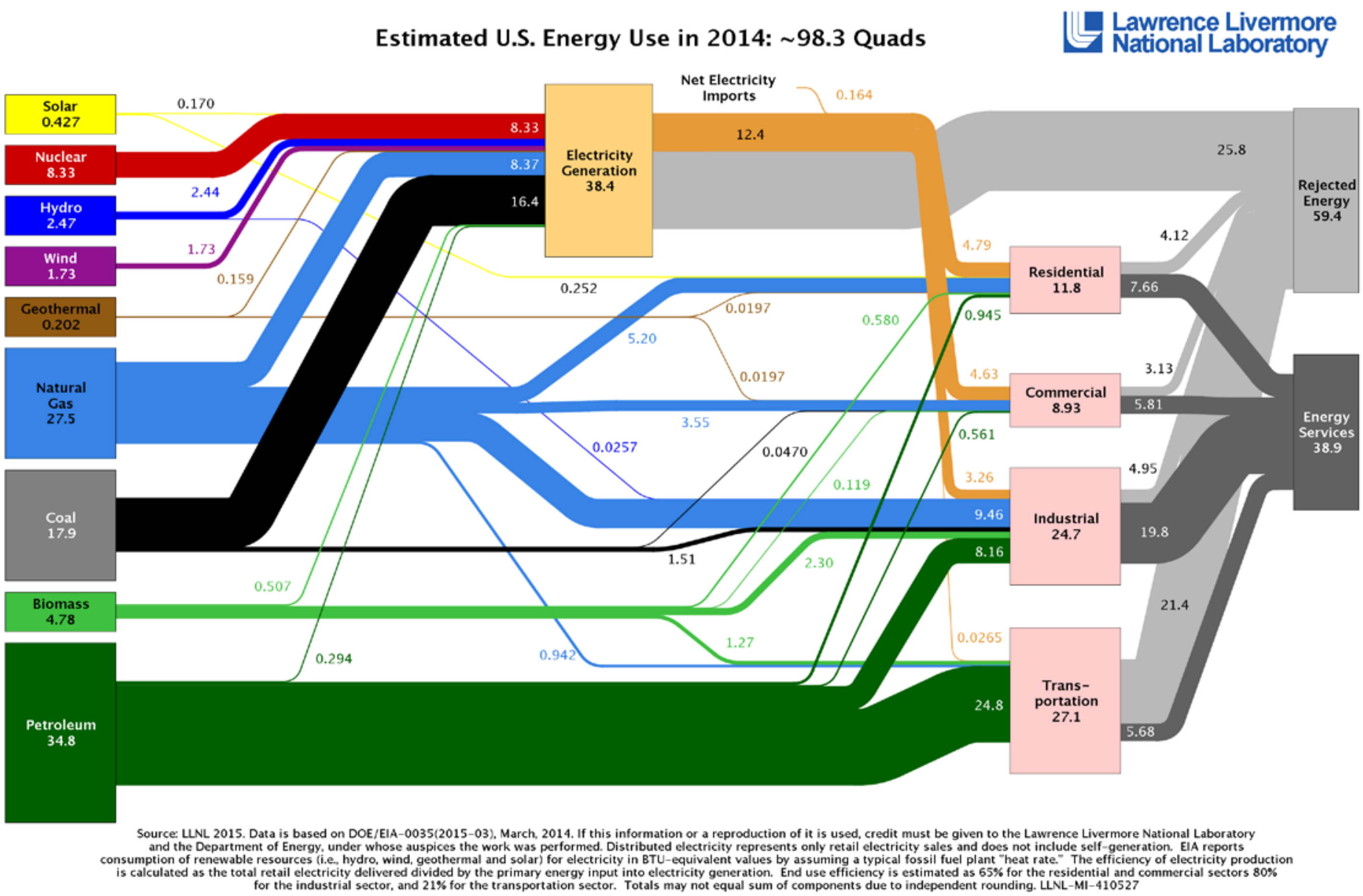
[59] https://en.wikipedia.org/wiki/Plutonium and
http://www.world-nuclear.org/info/nuclear-fuel-cycle/fuel-recycling/plutonium/
[60] http://www.world-nuclear.org/info/Nuclear-Fuel-Cycle/Introduction/Physics-of-Nuclear-Energy/
[61] http://www.nrdc.org/energy/renewables/solar.asp
[62] http://www.nrdc.org/energy/renewables/hydropower.asp
[63] http://www.nrdc.org/energy/renewables/wind.asp
[64] http://www.nrdc.org/energy/renewables/biomass.asp
[65] https://en.wikipedia.org/wiki/Cellulosic_ethanol

## Utilization of Energy Resources in the US in 2014[66]




"Rejected energy increased to 59 quads in 2013 from 58.1 in 2012, rising in proportion to the total energy consumed. 'Not all of the energy that we consume is put to use', [A. J.] Simon explained. 'Heat you feel when you put your hand on your water heater and the warm exhaust from your car's tailpipe are examples of rejected energy.' Comparing energy services to rejected energy gives a rough estimate of each sector's energy efficiency."[67]

Note: A *quad* is a unit of energy equal to $10^{15}$ BTU, or $1.055 \times 10^{18}$ joules (1.055 exajoules or EJ), or 293.08 Terawatt-hours (TWh).[68] The name quad derives from $10^{15}$ = 1 Peta = 1000 x (1,000**⁴**) = 1 short-scale quadrillion = 1 thousand trillion = 1 thousand thousand billion, etc. "Today, the United Kingdom officially uses the short scale [like the US], but France and Italy use the long scale."[69]

In 2010, 4125 TWh of electrical energy were generated in the US, mostly from fossil fuel, especially coal,[70] but the diagram above shows that natural gas by 2014 exceeds coal in quads of electrical energy production (reflecting increased gas production from fracking).

According to the US Energy Information Administration, world electricity generation in 2012 was 21,532 billion kWh, compared to 4,048 billion kWh for the US. Installed generating capacity in 2012 was 1,063 Million kW [1.063 TW] in the US versus 5,550 Million kW [5.550 TW] for the world. While the US has risen

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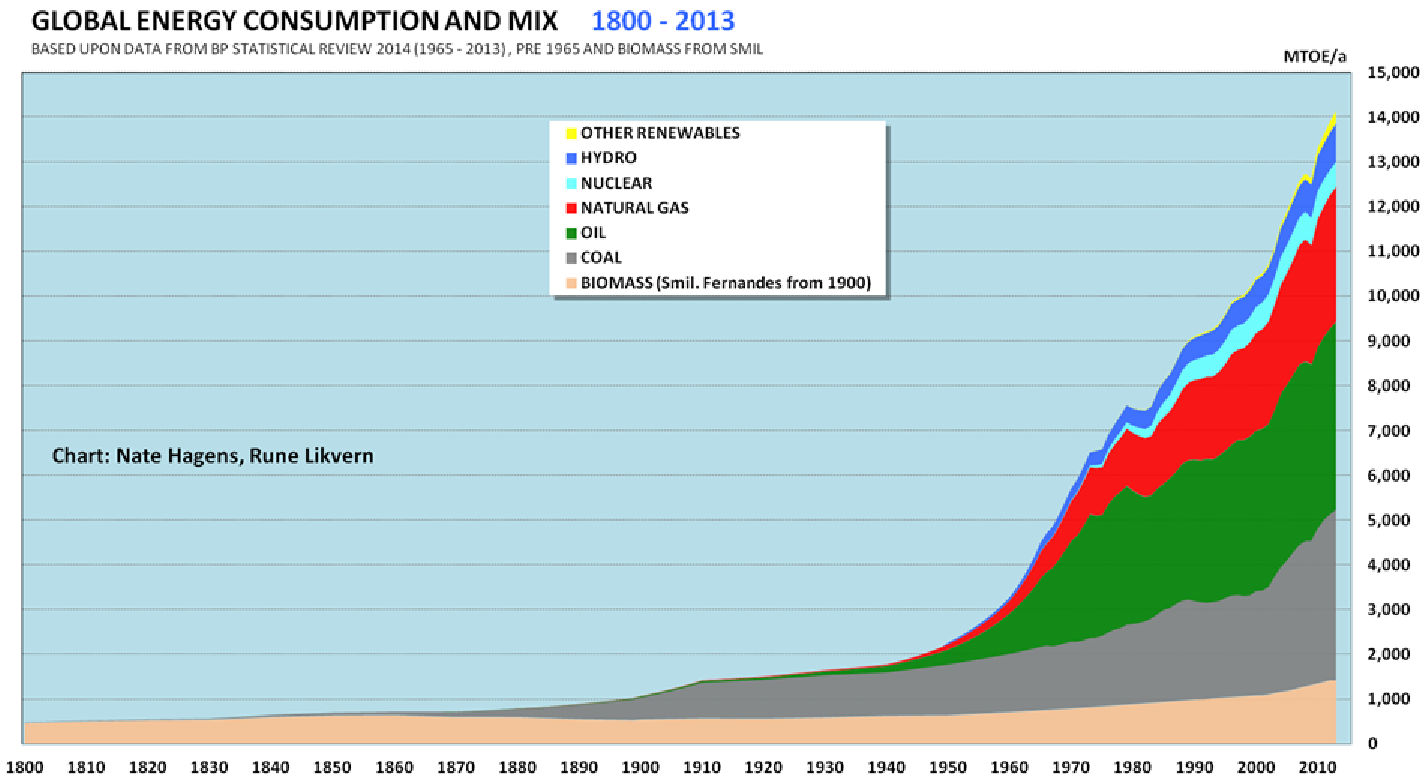
66 https://flowcharts.llnl.gov/
67 https://www.llnl.gov/news/americans-using-more-energy-according-lawrence-livermore-analysis
68 https://en.wikipedia.org/wiki/Quad_%28unit%29
69 https://en.wikipedia.org/wiki/Long_and_short_scales
70 *EEAI3* p. 41

only a little, Asia shows rapid rise in electricity generation, increasing from 4,469 billion kWh in 2002 to 8,762 in 2012. Total primary energy consumption for all types of energy for 2012 in the US was 95 quadrillion BTUs, compared to 524 for the entire world.[71]

The textbook author computes that per capita annual consumption of electricity in 2012 was 13.3 MWh for the US, 2.0 MWh for the remainder of the world, and 2.5 MWh for the whole world including the US. The high consumption in the US reflects not just high living standards but also an advanced industrial base.[72]

The following graph depicts global (world) energy consumption of all harnessed types from all sources (biomass, coal, oil, natural gas, nuclear, hydro, and other renewables) during the period 1800 to 2013. Clearly, total energy use has been rising nearly exponentially, most strikingly that of fossil fuels:[73]




The various forms of energy are here expressed in MTOE/a, that is Million Tonnes of Oil Equivalent. The /a in "MTOE/a" signifies total consumption (totaled for all humans)[74] rather than per capita consumption. One tonne of a substance is one metric ton or 1000 kg (approximately 2,205 lb., thus larger than a US ton of 2000 lb.). The tonne is confusingly slightly less than a UK ton (which is 2240 lb.).[75] One tonne of crude oil is said to release energy when burned of 41.9 GJ = 11.63 MWh = 39,683,207 BTU. One Mtoe represents $11.63 \times 10^6$ MWh. The final total peak value in the graph above of about 14,000 Mtoe represents $1.63 \times 10^{11}$ MWh annual equivalent consumption.

I will not attempt to summarize individual details about the various fossil fuels here. Many of the harmful effects of toxic byproducts are well known, including release of the greenhouse gas $CO_2$, CO, $SO_2$, $NO_x$, black carbon (soot), and carcinogenic and/or otherwise deleterious substances. These latter substances include benzene; petroleum coke (which contains toxic dusts with many compounds and heavy metals);

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[71] http://www.eia.gov/cfapps/ipdbproject/IEDIndex3.cfm?tid=2&pid=2&aid=12
[72] *EEAI3* p. 42-4
[73] http://fractionalflow.com/2014/10/10/the-powers-of-fossil-fuels/
[74] Robert Bent, Lloyd Orr, Randall Baker, *Energy: Science, Policy, and the Pursuit of Sustainability*, 2002, Island Press, p. 38
[75] https://en.wikipedia.org/wiki/Ton and https://en.wikipedia.org/wiki/Tonne

formaldehyde; polycyclic aromatic hydrocarbons (PAH); mercury; silica and other dusts; radon; and hydrofluoric acid HF, etc.[76]

The nuclear power industry also does not have an unblemished record, and environmental contamination with radioisotopes from reactors has been of great concern since Three-Mile Island (1979), Chernobyl (1986), and the Fukushima Daiichi nuclear disaster (2011).[77]

## Overall Electrical Generation in the US and World

The US EIA (U.S. Energy Information Administration)[78] provides these overall statistics for Electrical Generation, expressed in multiples of Watt-hours (Wh).

US Annual Totals

[Giga G = $10^9$, Tera T= $10^{12}$, Peta = $10^{15}$]
2002: 3,858 billion kWh = 3.858 x $10^3$ x $10^9$ x $10^3$ Wh = 3,858 x $10^{12}$ Wh = 3,858 TWh
2012: 4,048 billion kWh = 4.048 x $10^3$ x $10^9$ x $10^3$ Wh = 4,048 x $10^{12}$ Wh = 4,048 TWh

World Totals

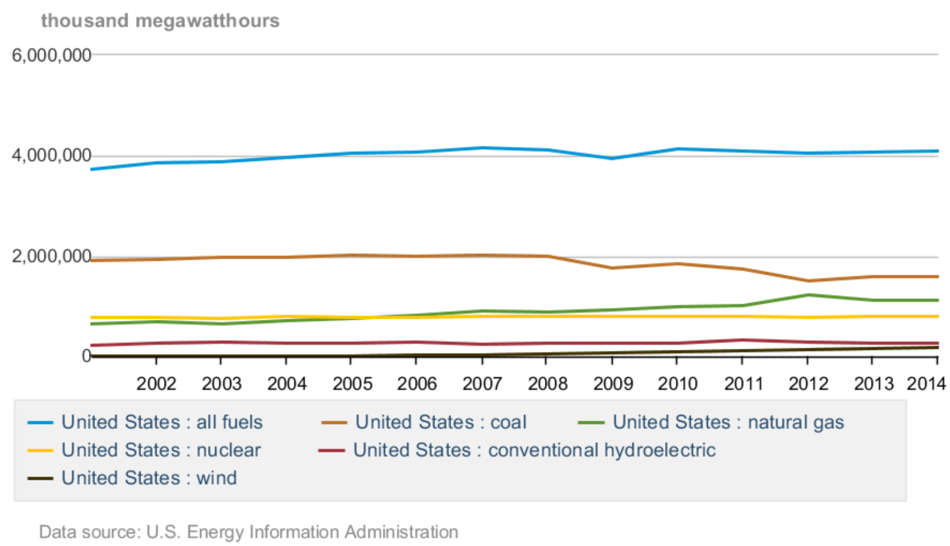
2002: 15,393 billion kWh = 15.393 x $10^3$ x $10^9$ x $10^3$ Wh = 15,393 x $10^{12}$ Wh = 15,393 TWh
2012: 21,532 billion kWh = 21.532 x $10^3$ x $10^9$ x $10^3$ Wh = 21,532 x $10^{12}$ Wh = 21,532 TWh

The first of the following graphs shows the EIA's statistics on electric power generation in the US, the graph extending from 2001 to 2014. The table show US electrical energy generation from 2009 to 2014. All values are expressed in thousands of MWh, thus in GWh. It is clear that most of US electrical power is generated from coal (though declining), nuclear energy (relatively constant), and natural gas (increasing).[79] Hydroelectric (fairly steady) and wind (increasing) make still small overall contributions.

The second table shows the EIA's statistics on electric power generation for the entire world, the data extending from 2008 to the most recently available year, 2012, and demonstrating how global electricity generation is steadily rising:

[76] http://planetsave.com/2013/12/07/pollution-air-pollution-water-pollution-health-problems/
[77] https://en.wikipedia.org/wiki/List_of_civilian_nuclear_accidents
[78] http://www.eia.gov/cfapps/ipdbproject/iedindex3.cfm
[79] http://www.eia.gov/electricity/data/browser and
http://www.eia.gov/cfapps/ipdbproject/iedindex3.cfm graphs and tables specified & modified by MCM

## Net generation from electricity plants for all sectors, annual




Units above are thousands of MWh = GWh

| Net generation from electricity plants for electric power ( | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|
| United States | | | | |
| All fuels | 3,948,186 | 3,890,358 | 3,903,715 | 3,935,968 |
| Coal | 1,717,891 | 1,500,557 | 1,567,722 | 1,571,868 |
| Petroleum liquids | 15,343 | 12,649 | 13,207 | 17,865 |
| Petroleum coke | 12,859 | 7,423 | 11,303 | 10,467 |
| Natural gas | 926,290 | 1,132,791 | 1,028,949 | 1,029,394 |
| Other gases | 2,939 | 2,984 | 4,322 | 3,944 |
| Nuclear | 790,204 | 769,331 | 789,016 | 797,067 |
| Conventional hydroelectric | 317,531 | 273,859 | 265,058 | 256,009 |
| Other renewables (total) | 163,886 | 188,081 | 221,461 | 248,350 |
| Wind | 120,121 | 140,749 | 167,742 | 181,643 |
| Solar | 1,727 | 4,164 | 8,724 | 17,869 |
| Geothermal | 15,316 | 15,562 | 15,775 | 16,628 |
| Biomass (total) | 26,722 | 27,606 | 29,220 | 32,209 |
| Wood and wood-derived fuels | 10,733 | 11,050 | 12,302 | 14,869 |
| Other biomass | 15,989 | 16,555 | 16,918 | 17,340 |
| Hydro-electric pumped storage | -6,421 | -4,950 | -4,681 | -6,209 |
| Other | 7,663 | 7,633 | 7,357 | 7,212 |

US Electrical Energy Generation 2002 to 2014 (units are thousands of MWh = GWh)

**International Energy Statistics** Electricity Generation

Country: World Start Year: 2008 End Year: 2012

Product: Total Net Generation Unit: Billion Kilowatthours

(Billion Kilowatthours)

| | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|
| World | 19,157.247 | 19,093.327 | 20,436.989 | 21,182.422 | 21,531.709 |

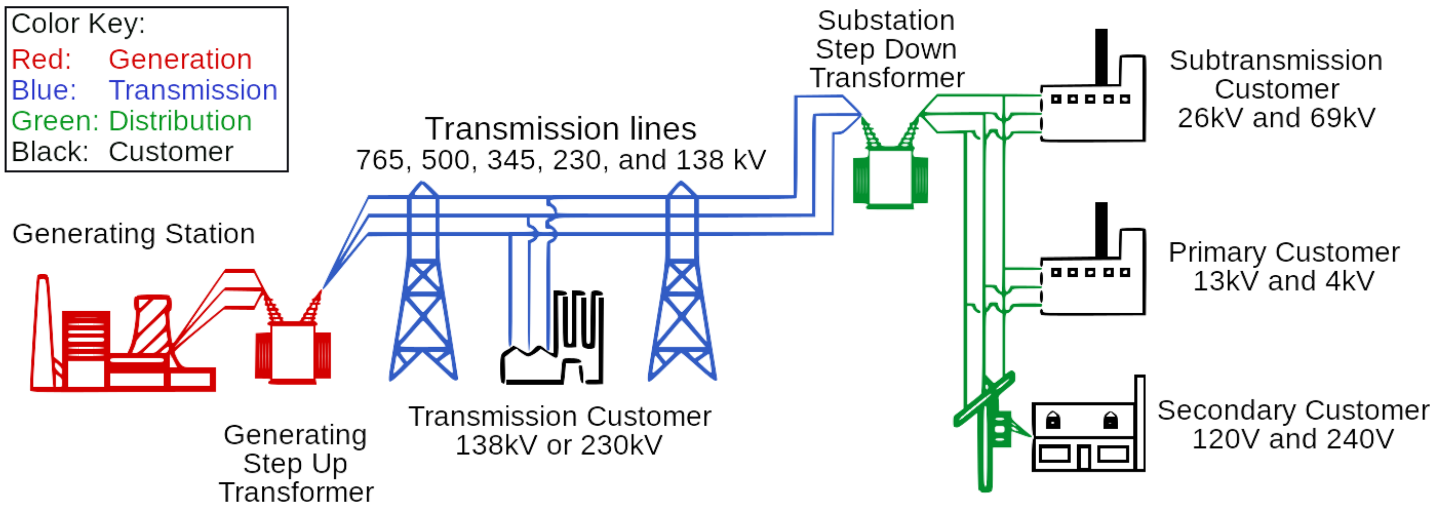
Total World Electrical Energy Generation 2008 to 2012 (units are TWh, note decimal point)

# Overview of AC Electrical Generation, Transmission and Distribution

Nikola Tesla conceived and championed our current AC electrical distribution system.

A modern diagram of our electrical system follows.[80] This shows or implies:

- original voltage generated at the electrical plant (typically 11 to 13 kV, 3-phase),
- step-up to high voltages (138 to 765 kV, 3-phase) by transmission transformer for long distance primary transmission, possibly with transmission voltage industrial customer in this voltage range,
- step-down to lower voltages (e.g., 26 to 69 kV, 3-phase) at distribution transformer for secondary transmission with possible subtransmission customer in this voltage range,
- step-down to 4 kV to 13 kV, 3-phase at distribution transformer for primary distribution, and
- step-down to 120 to 240 V at service transformer for distribution to residential customers (mostly single phase) and industrial or commercial customers (mostly 3-phase, voltages may be somewhat higher)




### Electrical Generation (discussed under individual modes of generation)

### Electrical Transmission (discussed in its own section below)

### Electrical Distribution

Electrical power is delivered to residences, businesses, etc. by the local *electric power company* (aka *power utility*, *energy service company*). For residences and small businesses, it is delivered via a *distribution system*, which includes primary distribution lines, *distribution substations* ,*distribution transformers* (pole mounted, pad mounted, or located inside a structure), and *secondary distribution* split-phase (single phase 120/240V) lines reaching the home (via an overhead *service drop* or underground *service lateral*),[81] etc. The customer's responsibility begins at the output of the electric meter. Much of the material pertaining to home electrical distribution is discussed below under Electrical Safety.

# Hydroelectric Power Plants

See also earlier tables on electrical generation in the US and the world. (This discussion draws in part on chapter 3 and chapter 4.) Adverse environmental effects of hydroelectric plants are mentioned briefly in chapter 5.[82]

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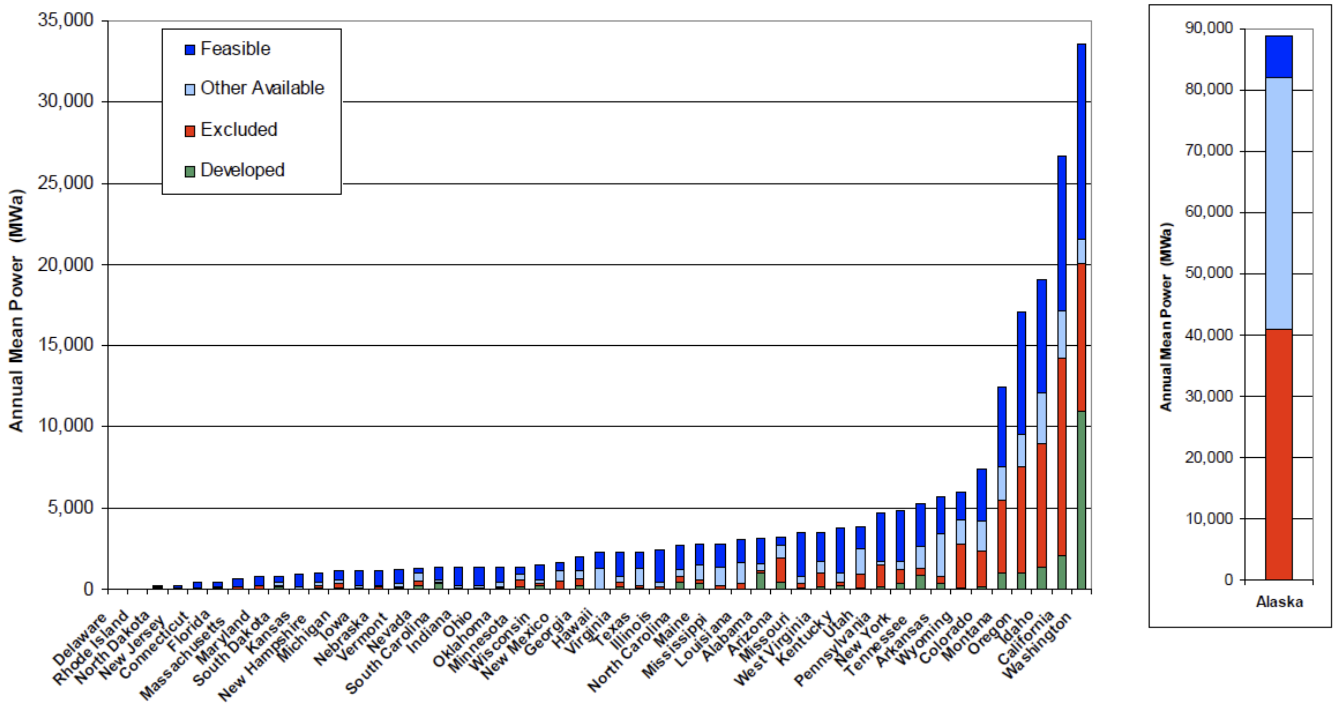
[80] https://en.wikipedia.org/wiki/Electricity_generation , see also this copyrighted image: http://www.electricaltechnology.org/2013/05/typical-ac-power-supply-system-scheme.html

[81] https://en.wikipedia.org/wiki/Service_drop

[82] *EEAI3* p. 96

## Hydroelectric Power Plant (HE PP) Capacity and Production

The following diagram shows Current Hydroelectric Capacity in the United States by state (developed, excluded, other, and feasible), showing the dominance of WA, CA, ID, and OR.[83]

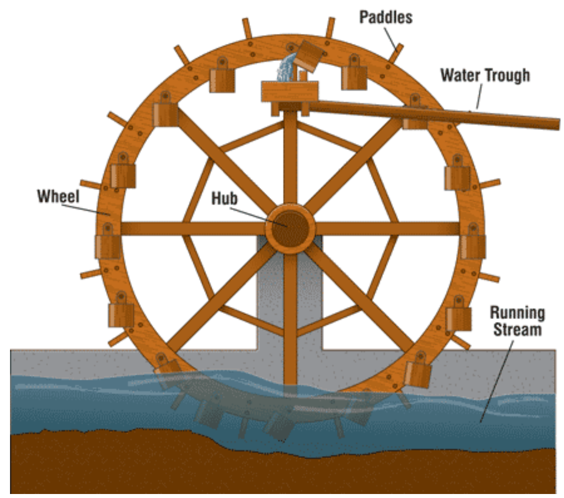



## Largest hydroelectric plants in the world (compared to selected US plants)

| | |
|---|---|
| Three Gorges, Yangtze R in Hubei Province, China: | 22.5 GW (completed 2010) |
| Itaipu (Paraná River, across the Brazil-Paraguay border): | 14.0 GW (completed 1983) |
| Guri (Venezuela) | 10 GW (completed 1986) |
| Grand Coulee, WA: | 6.8 GW (completed 1942) |
| Hoover, Colorado River, on the border between AZ and NV.: | 2.0 GW (generation began 1936) |

[83] https://wiki.uiowa.edu/display/greenergy/Hydroelectric+Power

## Terminology




**Hydropower**: To my way of thinking (and definitions vary), hydropower is water power, "power derived from the energy of falling water or fast running water".[84] It depends on the hydrologic cycle, in which the Sun evaporates sea and fresh water, the water precipitates as rain or snow, and the water flows in liquid form in rivers to return to the sea. *Hydropower* includes *hydroelectric power* (in fact some consider these terms synonymous)[85], but I prefer to regard it as a broader term that includes all forms of power harnessed from flowing water. Old technologies such mechanical rotation of a wheel by flowing river water (used in grinding mills, saw mills, and mechanical wheel water pumps such as the *noria*, depicted to the right[86]), power captured from river turbines,[87] tides and sea currents, as well as hydroelectric power, etc.

**Hydroelectric Power** specifically applies to conversion of the potential and/or kinetic energy of flowing water into electricity. The flow of water arises ultimately from solar energy, which is a renewable resource, but installed large-scale hydroelectric power generation in the US is relatively fixed in capacity currently, due to important environmental constraints. Thus, most authors do not consider it "renewable", at least in the US. Although the installed base in the US of HE PPs is relatively fixed and not expanding, new plants have been or will be built in recent decades in China (Three Gorges), Brazil/Paraguay (Itaipu), Vietnam, Ethiopia, etc. Small scale hydrokinetic power generation is considered renewable.

The earliest was built across the Fox River in Wisconsin, first operating in 1882.

## Types of HE PPs

1. **Impoundment HE PPs**

For example, Grand Coulee Dam. These typically generate the greatest amounts of electricity. They use a *dam* to create a lake or reservoir. Water under pressure feeds through one of more *penstocks* to *turbines* located at a lower level. A *governor* can regulate the rate of water flow presented to the turbines, and thus the power output, in order to match loads. Turbines are discussed further below. These can have a substantial environmental impact. .[88]

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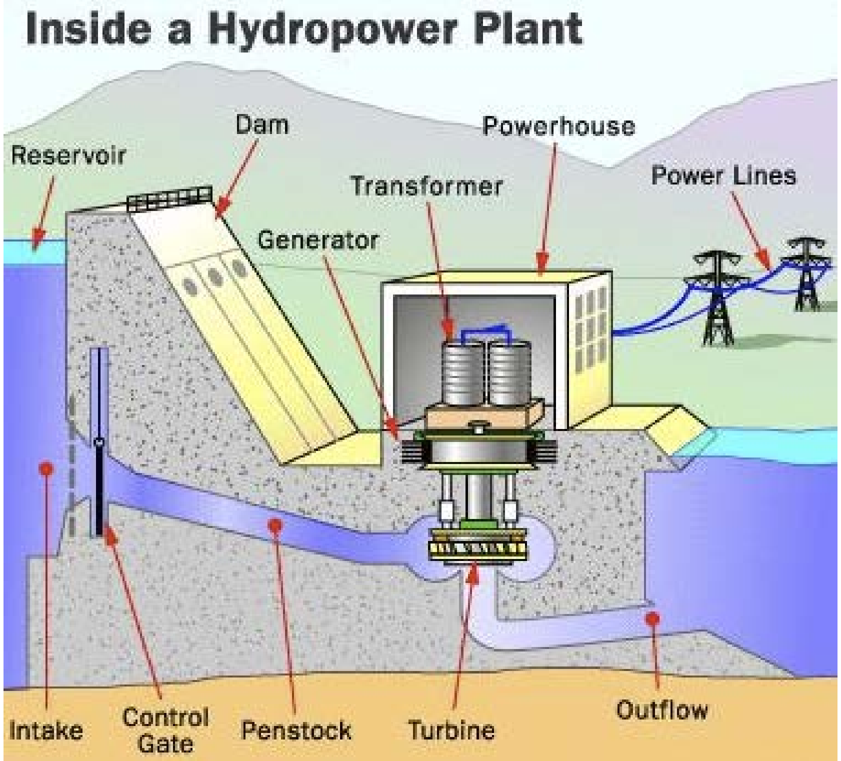
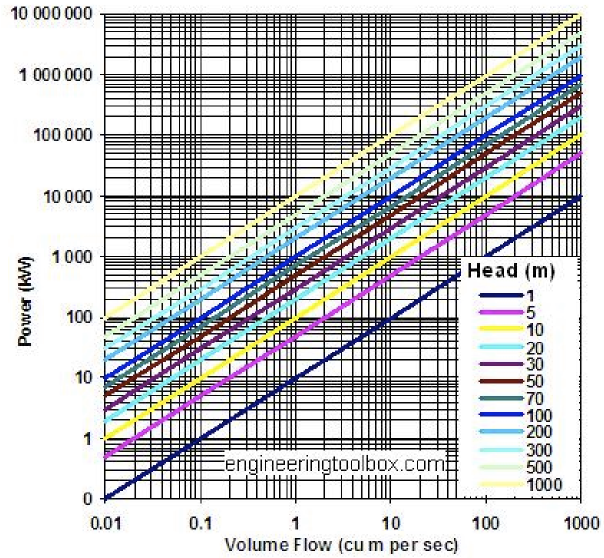
[84] https://en.wikipedia.org/wiki/Hydropower
[85] http://www.merriam-webster.com/dictionary/hydropower
[86] http://www.waterencyclopedia.com/Po-Re/Pumps-Traditional.html and
https://en.wikipedia.org/wiki/Noria
image from http://www.machinerylubrication.com/Read/1294/noria-history
[87] http://energy.gov/eere/water/water-power-program
[88] *EEAI3* p. 55, also http://www.usbr.gov/power/edu/pamphlet.pdf and




Impoundment HE PP. Many HE PPs have a dam, reservoir, and one or more sets of: control gate (aka governor), penstock (carrying water to the turbine), turbine connected to generator, and outflow channel.




Power generated in an Impoundment HE PP is a function of both water head and actual flow rate. The different colors represent different water pressures ("head" in m)

Both images are from this article[89]

When water flows in the penstock, the static pressure at the turbine inflow $P_{r0}$ (i.e., pressure for the blocked no flow condition) is reduced (by viscosity, frictional losses, and turbulence), so that the actual pressure at the turbine inflow $P_r < P_{r0}$ , and the head at the turbine is now designated the *effective head* h, where h < H. [90] In additional to penstock losses of power, there is power loss occurring at the conversion of water energy to turbine rotational energy, and in the generator's conversion of rotational energy to electrical output. Overall efficiency of power plant generation is given by the ratio of electrical power generated to PE + KE at the water intake. This ratio is given by

$$\frac{P_{out}}{P_{p-in}} = \eta_{total} = \eta_p \eta_h \eta_t \eta_g$$

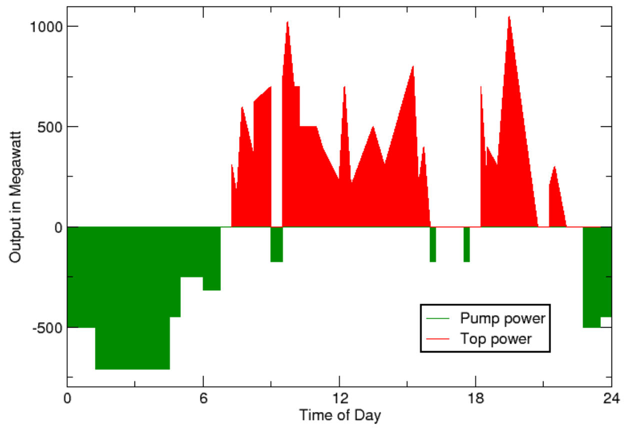
where

$\eta_p$ = penstock power transmission efficiency (0-1)
$\eta_h$ = penstock to turbine blade power conversion efficiency (0-1)
$\eta_t$ = turbine to generator power transmission efficiency (0-1), and
$\eta_g$ = generator mechanical to electrical power conversion efficiency (0-1).[91]

Estimates of overall electrical energy generation efficiency $\eta_{total}$ of modern HE PPs vary from as high as 80-90% or even 95% to as low as 50%-60% (the lower figures are especially applicable to small plants).[92] However, the sources of this information are not always clear as to exactly what computation is being used.

[89] https://wiki.uiowa.edu/display/greenergy/Hydroelectric+Power
[90] *EEAI3* p. 67
[91] *EEAI3* p. 69
[92] https://wiki.uiowa.edu/display/greenergy/Hydroelectric+Power
http://www.usbr.gov/power/edu/pamphlet.pdf 90%
http://www.wvic.com/Content/Facts_About_Hydropower.cfm 90%
http://www.mpoweruk.com/hydro_power.htm 95%
http://www.reuk.co.uk/Calculation-of-Hydro-Power.htm 50-60%

2. **Pumped Storage HE PPs**: These help with load balancing and more efficient power generation at HE installations. When electrical demand is low and extra electrical power is available, the power is used to pump water to an upper level water reservoir (or to raise the surface level of the reservoir). When electrical demand is high, the extra water is available to add to the head for additional power generation. The relatively low energy density of pumped storage systems requires either a very large body of water or a large variation in height, and this creates specific geographic constraint on suitable sites and can have a substantial environmental impact. The diagram shows a representative pattern at an unspecified site of pumping water (green) in off hours, and generating top power in times of higher demand.[93] In some cases, reversible Francis turbines are used for pumping.




3. **Diversion HE PPs**: These do not require a reservoir but utilize strong river currents to create a relatively low head that drives turbines for modest power output. E.g., Fox River in Wisconsin.

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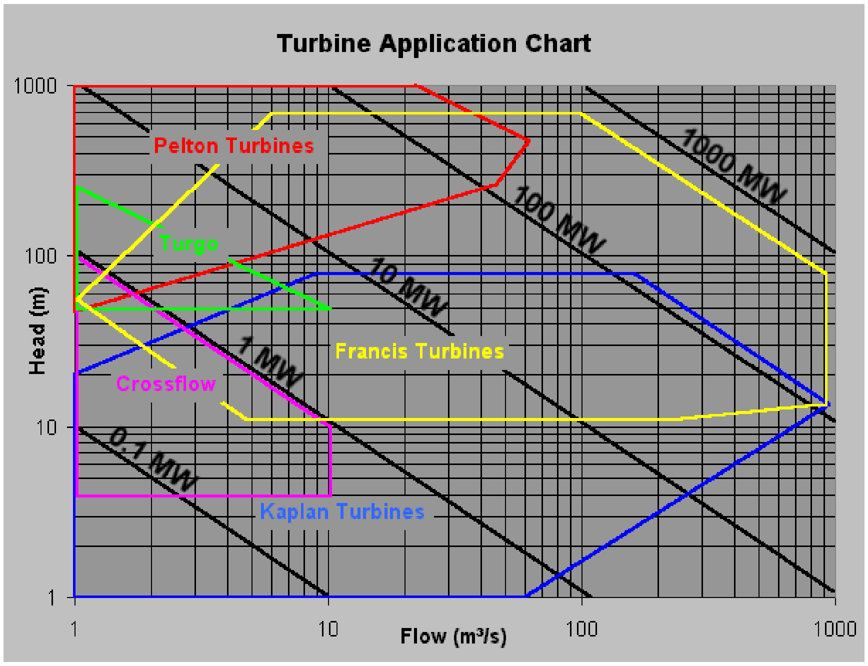
http://www.buzzle.com/articles/how-efficient-is-hydroelectric-power-generation.html 50% to 90%

[93] https://en.wikipedia.org/wiki/Pumped-storage_hydroelectricity and *EEAI3* p. 54.

## Categories of turbines and how they are selected

“An impulse turbine [e.g., Pelton] is generally suitable for high head, low flow applications… Reaction turbines are generally used for sites with lower head and higher flows than compared with the impulse turbines.”[94]

The following graph shows one author’s interpretation of optimal ranges for different types of turbine and thus how a particular type of turbine is chosen, namely: Francis, Pelton, and Kaplan, CrossFlow [impulse] and Turgo [impulse]. The parameters considered in the diagram are Head of pressure at the turbine (m) and Flow rate of water ($m^3/s$), as well as output megawatts of the turbine (or power plant?).[95]




## Impulse Turbines (mostly Pelton)

Impulse turbines operate on kinetic energy of water rather than pressure. They are mostly of the Pelton type, which utilize 1 or more jets of water directed in air against split buckets (cups, vanes) attached to the runner of the turbine. They are optimal for high head lower flow situations.

For Pelton turbines, the change of momentum of the water injected and reflected back in more-or-less the opposite direction yields a net linear force on the cups which may be theoretically as great as (approximately)[96]

$$F_c = 2\frac{m_i}{t}(v_i - v_c)$$

where $F_c$ = net force on a single cup in tangential direction (that of the jet)

$m_i/t$ = mass (kg) of water in the jet emitted per time interval t

$v_i$ = velocity of the incident jet relative to the cup

$v_c$ = linear velocity of the cup relative to the stationary enclosure.

As energy may be force x distance and power may be given by force times speed, the power acquired by the cup $P_c$ is

$$P_c = F_c v_c$$

It can be shown that maximum power is delivered when cup (runner) speed is 1/2 of the incident jet speed $v_i$.:

[94] http://energy.gov/eere/water/types-hydropower-turbines

[95] https://commons.wikimedia.org/wiki/File:Water_Turbine_Chart.png by anon. author “Tonigonenstein”

[96] All computations for Pelton turbines are from *EEAI3* p. 57-61

$$v_i = v_c$$

in which case, maximum power captured is

$$P_{c-max} = \frac{m_i}{t} v_i^2$$

In this maximum power capture condition, the full KE of the incident jet is captured.

Expressing power in terms of volume flow rate rather than mass flow rate,

$$P_c = 2\frac{vol_i}{t}\delta(v_i - v_c)v_c = 2f\delta(v_i - v_c)v_c$$

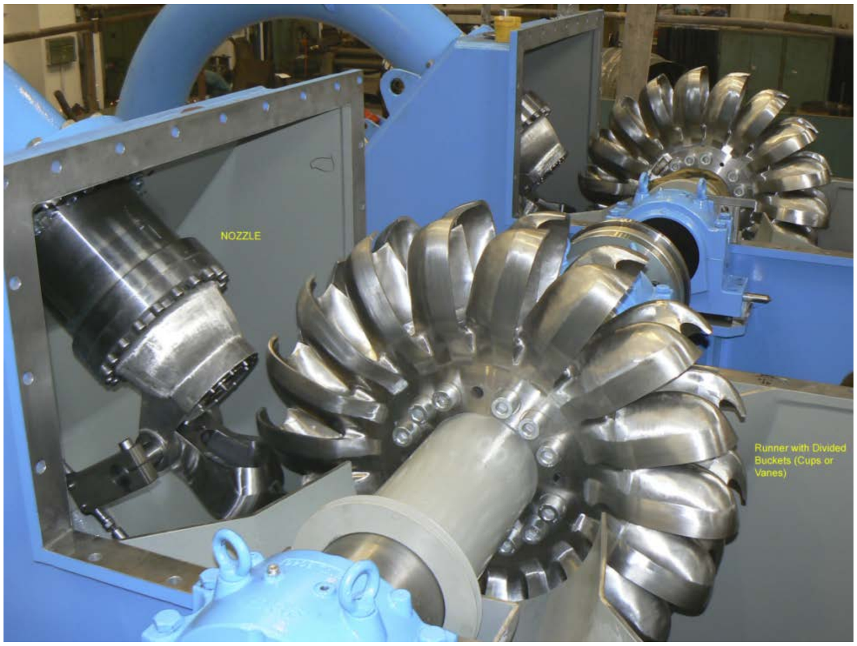
where f = volume flow rate in jet ($m^3/s$) = $Av_i$ (where A is cross section of jet)
δ = water density $kg/m^3$

The maximum power is delivered when

$$P_{c-max} = \frac{1}{2} f\delta v_i^2$$

Pelton turbines are depicted in the following two images:

Shown above is a horizontally mounted Pelton turbine showing 5 injection jets within the turbine casing.




Shown above is a vertically mounted Pelton turbine runner with stainless steels cups and a single adjacent jet nozzle. The turbine casing has been removed.[97]

Other types of impulse turbine include the *Cross-Flow*[98] and *Turgo*.[99]

### Reaction Turbines

These are completely immersed in water and are said to operate more on pressure rather than kinetic energy. However, the Francis turbine is said to combine impulse and reaction characteristics. According to the textbook *EEAI3*, Francis are suitable for 80 to 500 m heads, whereas Kaplan are suitable for lower heads of 1.5-80m.

#### Francis Turbine

Francis turbines are high efficiency, operate well over a wide range of operating conditions and are widely used in HE PP, contributing 60% of global hydropower capacity. The fixed *blades* (aka buckets or vanes) are complexly shaped like airfoils, and experience both an impulse as well as a lift force (via Bernoulli effect). These are thus mixed impulse and reaction turbines. The water enters the spinning blades more or less radially and exits below axially. Flow in the *spiral casing (scroll case)* surrounding the blades has continuously decreasing cross section, so that as water is directed into the turbine blades, the cross sectional decrease keeps the water flow velocity nearly uniform. *Stay vanes* (fixed vanes, aka wicket gates) and *guide vanes* (fixed in vertical axis but having adjustable variable angle) redirect flow toward the rotating blades. The variable guide vanes are used to control how much water is injected and thus can control power output of the turbine and therefore the generator to match power demand. They also control inlet flow angles to keep the angle of attack optimal.

The low pressure side where water exits below is subject to cavitation (because local pressure at the exit side drops below the saturation vapor pressure of water) and severe erosional damage can result. Thus, the exiting *draft tube* requires careful design attention, including a gradually increasing cross-sectional area for

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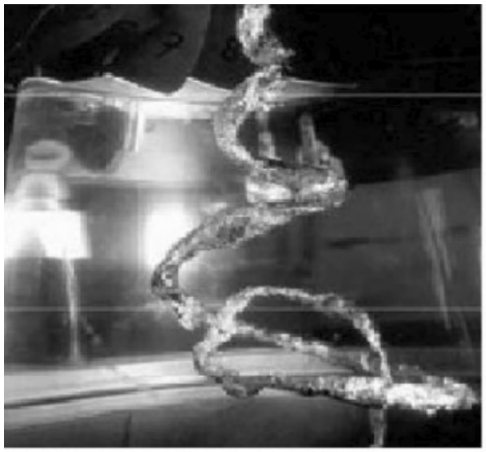
[97] http://www.hydrolink.cz/en/pelton-turbines/hhp-h-type-horizontal-compact-pelton-turbine-4.html ,
both images extracted from this article and the 2nd was modified slightly by MCM

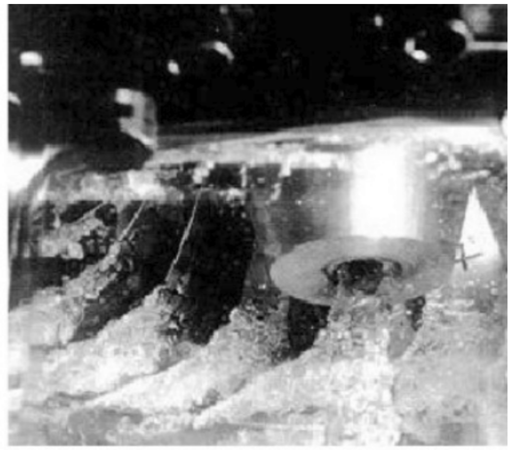
[98] http://energy.gov/eere/water/types-hydropower-turbines

[99] https://en.wikipedia.org/wiki/Turgo_turbine

gradually transitioning from velocity head to static head.[100] The following images depict several types of cavitation in rotating Francis turbines.[101]

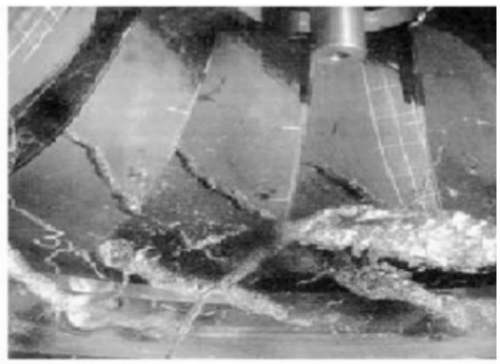


*(a) Draft tube swirl*



*(b) Travelling edge cavitation*

*(c) Leading edge cavitation*



*(d) Inter-blade vortex cavitation*

Assuming pressure energy is the dominant component (and therefore that KE can be neglected), the energy imparted to the blades is given by:

$$\mathrm{E_{blades}} = (\mathrm{P_{r1}} - \mathrm{P_{r2}})\mathrm{vol} + \frac{1}{2}\mathrm{m}(\mathrm{v}_1^2 - \mathrm{v}_2^2) \approx (\mathrm{P_{r1}} - \mathrm{P_{r2}})\mathrm{vol} \approx \mathrm{P_{r1}}\mathrm{vol}$$

where $P_{r1}$ is the pressure just before encountering the blades
$P_{r2}$ is the pressure just after encountering the blades (assumed to be very low)
$v_1$ is the water velocity just before encountering the blades
$v_2$ is the water velocity just after encountering the blades
vol is the volume of water causing the energy deposition

The power imparted to the blades is therefore

$$\mathrm{P}_{blades} \approx \mathrm{P_{r1}}\frac{\mathrm{vol}}{t} \approx P_{r1}f$$

where f is the penstock flow rate in $m^3/s$

[100] http://www.learnengineering.org/2014/01/how-does-francis-turbine-work.html etc.
[101] http://www.slideshare.net/saurabh856/cavitation-in-francis-turbine


Runner with blades of a Francis turbine installed in the Sanxia (Three Gorges Dam) power plant in Yichang, Hubei province. There are 32 main turbine/generator units each at capacity of 700-710 MW (plus two plant power generators, each with capacity of 50 MW), and total installed capacity of 22,500 MW. This single turbine and its generator have the following specifications:[102]

| | |
|---|---|
| ▪ Turbine Rotational Speed (rpm) | 71.4/75 |
| ▪ Nominal net head (m) | 80.6 |
| ▪ Runner weight (t) | 46.3 |
| ▪ Runner diameter (m) | 9.6 |
| ▪ Generator Stator Bore (m) | 18.8 |
| ▪ Rotor Weight (t) 2,000 | 2,000 |
| ▪ Capacity (MW) | 700-710 |

## Kaplan Turbine

> "The Kaplan turbine is a propeller-type water turbine which has adjustable blades. It was developed in 1913 by the Austrian professor Viktor Kaplan, who combined automatically adjusted propeller blades with automatically adjusted wicket gates to achieve efficiency over a wide range of flow and water level... Its invention allowed efficient power production in low-head applications that was not possible with Francis turbines. The head [of pressure] ranges from 10–70 meters and the output from 5 to 200 MW. Runner diameters are between 2 and 11 meters. The range of the turbine rotation is from 79 to 429 rpm... Kaplan turbines are now widely used throughout the world in high-flow, low-head power production... Power is recovered from both the hydrostatic head and from the kinetic energy of the flowing water. The design combines features of radial and axial turbines. The inlet is a scroll-shaped tube that

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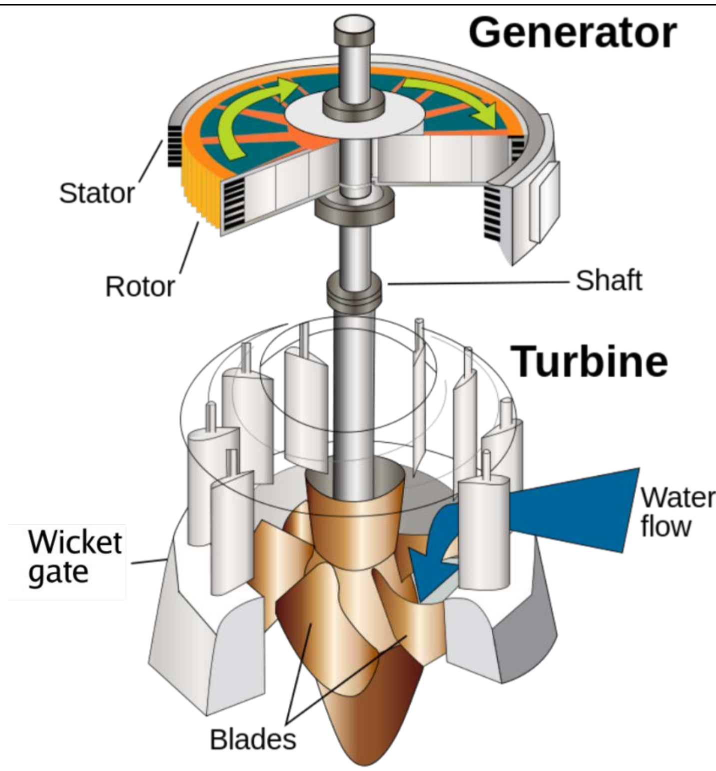
[102] http://www.alstom.com/Global/Power/Resources/Documents/Brochures/three-gorges-hydro-power-plant-china.pdf , and
https://en.wikipedia.org/wiki/Three_Gorges_Dam

wraps around the turbine's wicket gate. Water is directed tangentially through the wicket gate and spirals on to a propeller shaped runner, causing it to spin… Kaplan turbines are widely used throughout the world for electrical power production. They cover the lowest head hydro sites and are especially suited for high flow conditions… Inexpensive micro turbines on the Kaplan turbine model are manufactured for individual power production designed for 3 m of head, [but] can work with as little as 0.3 m of head at a highly reduced performance provided [there is] sufficient water flow… Large Kaplan turbines are individually designed for each site to operate at the highest possible efficiency, typically over 90%. They are very expensive to design, manufacture and install, but operate for decades."[103] These turbines may be installed with vertical, horizontal, or oblique axis.[104] Kaplan turbines are now widely used throughout the world in high-flow, low-head power production.[105]

The Manuel Piar Hydroelectric Power Plant (Tocoma Dam) in Venezuela, scheduled to open in 2015, has Kaplan turbines that generate the greatest power at nominal head as of 2012. The total installed capacity is 2,300 megawatts at rated head 34.65 m. This capacity derives from ten Kaplan generator units manufactured by IMPSA, each producing 230 megawatts. The diameter of the runner is 8.6 metres (28 ft).[106]




Vertical axis Kaplan turbine showing wicket gates which direct water against the variable pitch airfoil-like blades, and shaft connection to generator rotor. Water exits below in this case.[107]

## Fossil Fuel Power Plants

Fossil Fuel Power Plants are discussed quite briefly in the textbook in chapter 4[108] and also briefly here. I have discussed the relevant thermodynamic cycles and steam turbine technology in a separate summary.[109]

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103 https://en.wikipedia.org/wiki/Kaplan_turbine

104 http://www.koessler.com/sites/default/files/produkte/Anlagebl%25C3%25A4tter%2520Kaplan%25202013%2520Englisch.pdf

105 http://www.hydroquebec.com/learning/hydroelectricite/types-turbines.html

106 Paraphrased from http://www.impsa.com/en/downloads/HYDRO/TOCOMA.pdf and https://en.wikipedia.org/wiki/Tocoma_Dam

107 https://commons.wikimedia.org/wiki/File:Water_turbine_%28en_2%29.svg

108 *EEAI3* p. 70-75

The heavy environmental impacts of fossil fuels, including atmospheric pollution from $CO_2$, $SO_2$, $NO_x$, acid rain, and ozone, as well as ashes, Legionella transmission from cooling towers, etc., are mentioned only briefly in Chapter 5.[110] I have provided a much more thorough summary pertaining to atmospheric pollution in a separate document.[111]

Such plants depend on heat generated from burning fossil fuels (coal, natural gas, petroleum liquids—diesel, fuel oil, etc.—and (solid) petroleum coke). The amount of electrical power generated by these plants compared to hydroelectric and other sources of electrical power generation are shown in the earlier table, "US Electrical Energy Generation 2002 to 2014".

## Thermal (Thermodynamic) Cycle

Because they depend on a thermal (thermodynamic) cycle, the conversion to electrical energy by burning fuel in fossil fuel plants is quite inefficient. The *heat sink* (at $T_{Low}$, usually a cooling tower) must extract and release *waste heat* into the environment, in order for the thermal cycle to succeed.

The following diagram[112] illustrates an idealized Rankine thermodynamic cycle using water/steam as the working fluid and with simplified and suboptimal operating characteristics.[113] Here, $\dot{Q}$ = heat flow rate, $\dot{W}$ = power consumed by (pump) or provided to the system (turbine).

**Process 1-2, Isentropic compression by a pump:** The working fluid is pumped from low to high pressure (at state 2). Because the fluid is a liquid at this stage (it lies to the left of the saturated liquid line in the compressed liquid region), the pump requires relatively little input energy.

**Process 2-3, Constant pressure heat addition in a boiler**: The high pressure compressed liquid enters a boiler where it is heated at constant pressure by an external heat source to become first a saturated liquid, then a saturated liquid-vapor mixture, then a superheated vapor (ending at 3, a subcritical state having $T_3 < T_{cr}$). The input energy $Q_{in}$ required to attain this state can be calculated graphically, using an enthalpy-entropy chart (aka h-s or Mollier diagram), or numerically, using steam tables.

**Process 3-4, Isentropic expansion in a turbine**: The superheated vapor expands through a turbine, generating power. This decreases the temperature and pressure of the vapor, and some condensation may occur (as implied in the diagram lower left), though at state 4, there is and should be at most minimal condensation. The turbine net work output

$$W_{net,out} = W_{out} - W_{in}$$

for this process (i.e., net after allowing for pump $Work_{in}$) can be easily calculated using the charts or tables.

**Process 4-1, Constant pressure heat rejection in a condenser**: The slightly wet vapor at 4 then enters a condenser where it is condensed at a constant pressure to become a saturated liquid-vapor mixture, and finally a saturated liquid (at 1). Heat $Q_{out}$ is given off as a result of the enthalpy of vaporization resulting from condensation of the vapor.

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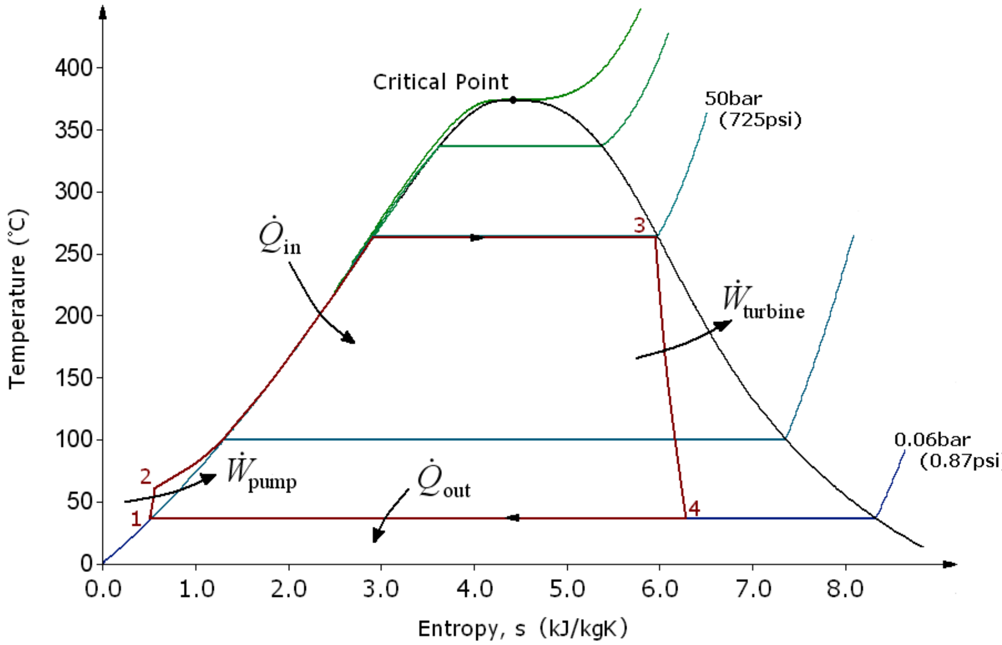
109 http://www.mcgoodwin.net/pages/thermodynamics.pdf
110 *EEAI3* p. 89-95
111 http://www.mcgoodwin.net/pages/atmscuw458.pdf
112 https://en.wikipedia.org/wiki/Rankine_cycle , image uploaded by https://en.wikipedia.org/wiki/User:Andrew.Ainsworth , who also provides the annotation.
113 I have discussed Rankine cycle thermodynamics in much greater detail in http://www.mcgoodwin.net/pages/thermodynamics.pdf




Idealized Rankine thermodynamic cycle with simplified and suboptimal operating characteristics:

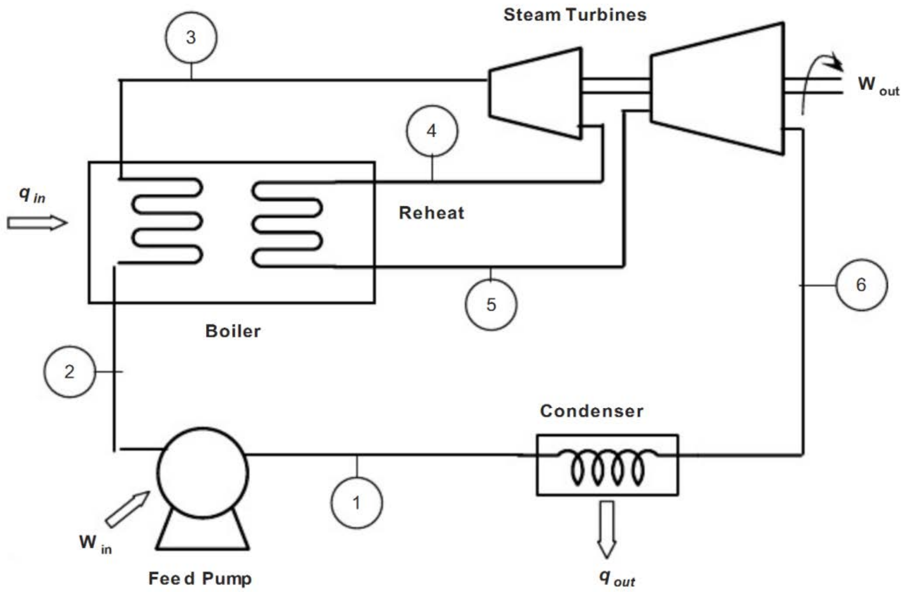
> "T-s (Temperature vs. specific entropy) diagram of a basic Rankine cycle using water/steam in SI units. Data derived from IAPWS IF-97 [The International Association for the Properties of Water and Steam]... Isobars are at pressures 0.06 bar, 1.01325 bar (1atm), 50 bar, 150 bar and 221 bar. Temperature rise associated with pump is heavily exaggerated for clarity and cycle operates between pressures of 50 bar and 0.06 bar...Easiest to think of the cycle starting at the pump and so input to pump is state 1."

Actual cycles deviate from the ideal:

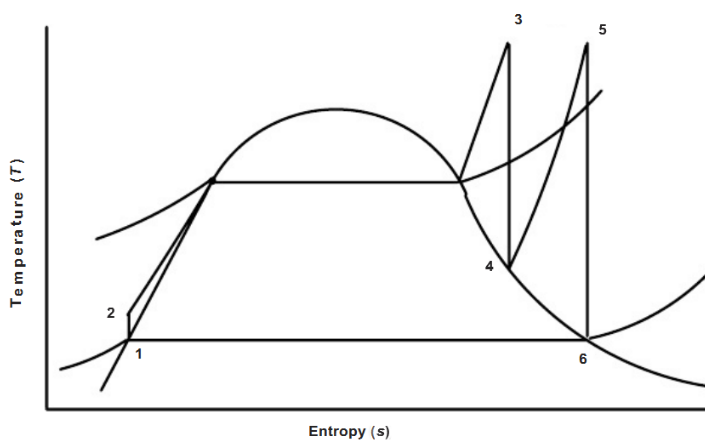
> "The compression by the pump and the expansion in the turbine are not isentropic. In other words, these processes are non-reversible and entropy is increased during the two processes. This somewhat increases the power required by the pump and decreases the power generated by the turbine."[114]

[114] https://en.wikipedia.org/wiki/Rankine_cycle , image uploaded by https://en.wikipedia.org/wiki/User:Andrew.Ainsworth , who also provides the annotation.

A slightly more complex Rankine cycle with superheating and reheating is depicted to follow:




The corresponding Rankine cycle with reheat follows:[115]




The $T$-$s$ diagram of a reheat Rankine cycle.

> "The purpose of a reheating cycle is to remove the moisture carried by the steam at the final stages of the expansion process [in the turbine]. In this variation, two turbines work in series. The first [High Pressure HP Turbine] accepts vapor from the boiler at high pressure. After the vapor has passed through the first turbine, it re-enters the boiler and is reheated before passing through a second, lower-pressure, turbine. The reheat temperatures [at 5] are very close or equal to the inlet temperatures [at 3], whereas the optimum reheat pressure needed [along 4-5] is only one fourth of the original boiler pressure [attained at 3]. Among other advantages, this prevents the vapor from condensing during its expansion [in the turbine] and thereby damaging the turbine blades, and improves the efficiency of the cycle, because more of the heat flow [in the turbine] occurs at higher temperature… The idea behind double reheating is to increase the average temperature. It was observed that more than two stages of reheating are unnecessary, since the next stage increases the cycle efficiency only half as much as the preceding stage. Today, double reheating is commonly used in power plants that operate under supercritical pressure [the diagram does not depict supercritical pressure]."[116]

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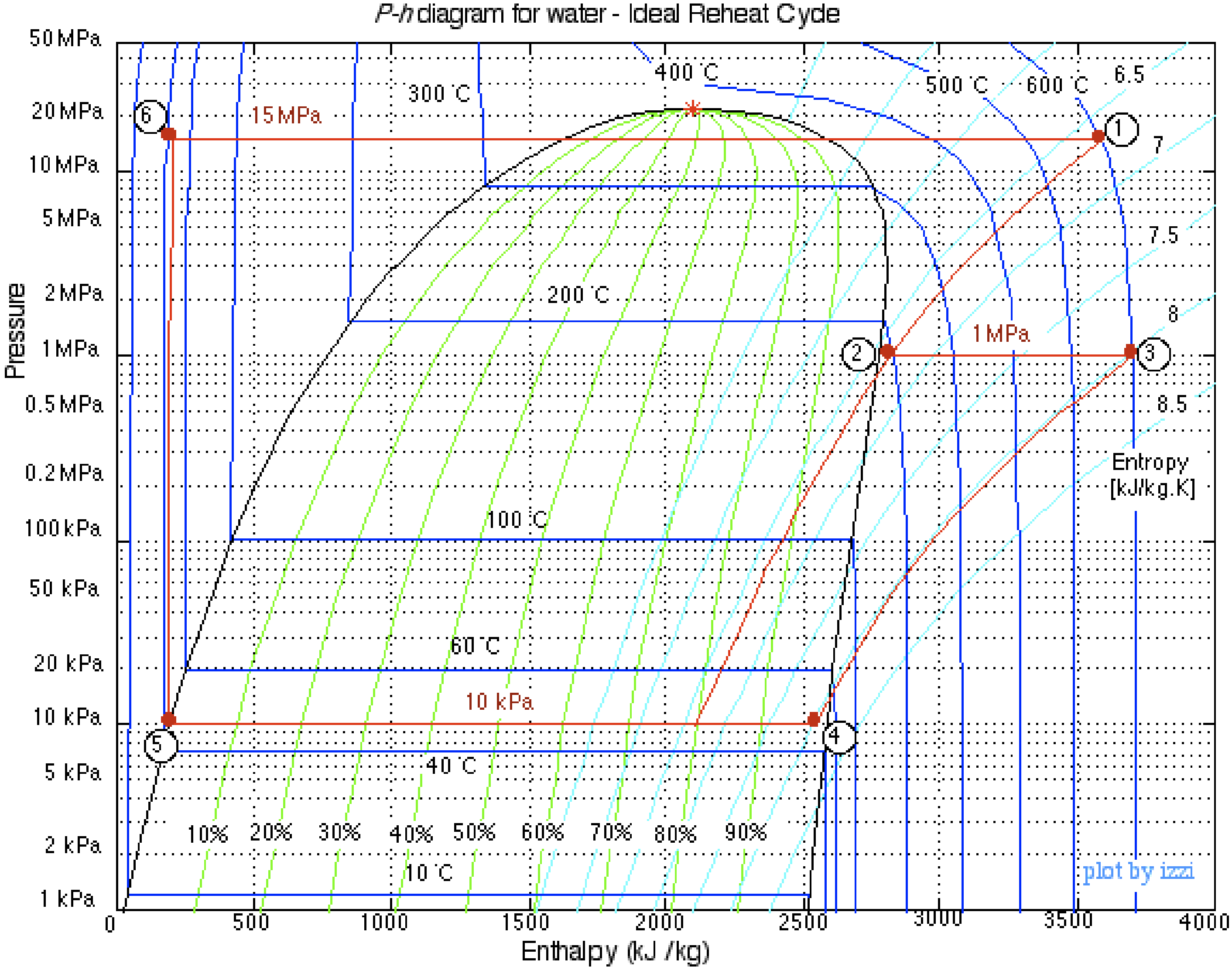
115 https://www.scribd.com/doc/54610589/10/Combined-Reheat-and-Regenerative-Rankine-Cycle (both images)
116 https://en.wikipedia.org/wiki/Rankine_cycle

The following pressure - specific enthalpy (P-h) diagram[117] depicts the operation of an idealized Rankine cycle that is more optimal. It is operating in subcritical pressures and temperatures, and it incorporates superheating [at states 1 and 3] and reheating [in process 2-3]. The cycle depicted also shows that the ending turbine state [4] is associated with only slight condensation under the *saturation dome*, preventing erosion of the blades:




[117] https://www.ohio.edu/mechanical/thermo/Applied/Chapt.7_11/Chapter8a.html

## Types of Turbines (aka "Prime Movers") used in Thermal Power Plants

Prime Movers (including Turbines) "… are typically Diesel Engines, Gas or Steam Turbines, or Hydro and Wind Turbines. Prime movers convert oil, gas, coal, wood, uranium, water, wind, etc. into mechanical energy. The mechanical [rotational] energy is supplied to the shaft of the generator "[118]

I have made an extensive summary here[119] of the thermodynamics and operation of steam turbines (vapor power cycle plants), gas turbines (gas power cycles plants), combined cycle plants, and other power cycles used in power generation. This information includes discussion of thermal efficiencies.

### Steam Turbines

Steam turbine plants use the dynamic pressure generated by expanding steam to turn the blades of a turbine. Almost all large non-hydro plants use this system, and they are the focus of most of this section.[120]

### Gas Turbines

Gas turbine plants use the dynamic pressure from flowing gases (air and combustion products) to directly operate the turbine. They are often natural gas or oil fueled.

A major selling point for the gas turbine power plant is that it operates at very high temperatures, thus should be thermodynamically more efficient. (It can also be very clean burning.) The following is from the Office of Fossil Energy:

> "The combustion (gas) turbines being installed in many of today's natural-gas-fueled power plants are complex machines, but they basically involve three main sections:
>
> • The *compressor*, which draws air into the engine, pressurizes it, and feeds it to the combustion chamber at speeds of hundreds of miles per hour.
>
> • The *combustion system*, typically made up of a ring of fuel injectors that inject a steady stream of fuel into combustion chambers where it mixes with the air. The mixture is burned at temperatures of more than 2000 degrees F. The combustion produces a high temperature, high pressure gas stream that enters and expands through the turbine section.
>
> • The turbine is an intricate array of alternate stationary and rotating aerofoil-section blades. As hot combustion gas expands through the turbine, it spins the rotating blades. The rotating blades perform a dual function: they drive the compressor to draw more pressurized air into the combustion section, and they spin a generator to produce electricity.
>
> Land based gas turbines are of two types:
> (1) **Heavy frame engines** are characterized by lower pressure ratios (typically below 20) and tend to be physically large. Pressure ratio is the ratio of the compressor discharge pressure and the inlet air pressure. Aeroderivative engines are derived from jet engines, as the name implies, and operate at very high compression ratios (typically in excess of 30).
> (2) **Aeroderivative engines** tend to be very compact and are useful where smaller power outputs are needed. As large frame turbines have higher power outputs, they can produce larger amounts of emissions, and must be designed to achieve low emissions of pollutants, such as NOx.
>
> One key to a turbine's fuel-to-power efficiency is the temperature at which it operates. Higher temperatures generally mean higher efficiencies, which in turn, can lead to more economical operation. Gas flowing through a typical power plant turbine can be as hot as 2300 degrees F, but some of the critical metals in the turbine can withstand temperatures only as hot as 1500 to 1700 degrees F. Therefore, air from the compressor might be used for cooling key turbine components, reducing ultimate thermal efficiency.

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[118] http://electricalengineeringtour.blogspot.com/2012/08/how-prime-movers-works-in-electic-power.html syntax corrected MCM
[119] http://www.mcgoodwin.net/pages/thermodynamics.pdf
[120] https://en.wikipedia.org/wiki/Power_station

> One of the major achievements of the Department of Energy's advanced turbine program was to break through previous limitations on turbine temperatures, using a combination of innovative cooling technologies and advanced materials. [These advanced turbines] … were able to boost turbine inlet temperatures to as high as 2600 degrees F - nearly 300 degrees hotter than in previous turbines, and achieve efficiencies as high as 60 percent.
>
> Another way to boost efficiency is to install a *recuperator* or *heat recovery steam generator (HRSG)* to recover energy from the turbine's exhaust. A recuperator captures waste heat in the turbine exhaust system to preheat the compressor discharge air before it enters the combustion chamber. A HRSG generates steam by capturing heat from the turbine exhaust. These boilers are also known as *heat recovery steam generators*. High-pressure steam from these boilers can be used to generate additional electric power with steam turbines, a configuration called a *combined cycle*.
>
> A simple cycle gas turbine can achieve energy conversion efficiencies ranging between 20 and 35 percent. With the higher temperatures achieved in the Department of Energy's turbine program, future hydrogen and syngas fired gas turbine combined cycle plants are likely to achieve efficiencies of 60 percent or more. When waste heat is captured from these systems for heating or industrial purposes, the overall energy cycle efficiency could approach 80 percent."[121]

(But see below about actual average Heat Rate data currently achieved.)

Additional technologies which have been implemented or hold promise for gas turbines include:

> (1) hydrogen turbines[122] which use hydrogen as fuel, some of which may derive from coal[123]
> (2) coal gasification,[124] which forms a mixture of carbon monoxide, hydrogen and other gaseous compounds that can be burned

The newer gasification technologies also are aimed at reducing *atmospheric pollution* by NOx, SOx, and particulates:[125]

> "The environmental benefits of gasification stem from the capability to achieve extremely low SOx, NOx and particulate emissions from burning coal-derived gases. Sulfur in coal, for example, is converted to hydrogen sulfide and can be captured by processes presently used in the chemical industry. In some methods, the sulfur can be extracted in either a liquid or solid form that can be sold commercially. In an *Integrated Gasification Combined-Cycle (IGCC)* plant, the *syngas* produced is virtually free of fuel-bound nitrogen [i.e., N found naturally in coal]. NOx from the gas turbine is limited to thermal NOx. Diluting the syngas allows for NOx emissions as low as 15 parts per million. *Selective Catalytic Reduction (SCR)* can be used to reach levels comparable to firing with natural gas if required to meet more stringent emission levels. Other advanced emission control processes are being developed that could reduce NOx from hydrogen fired turbines to as low as 2 parts per million.
>
> The Office of Fossil Energy is also exploring advanced syngas cleaning and conditioning processes that are even more effective in eliminating emissions from coal gasifiers. Multi-contaminant control processes are being developed that reduce pollutants to parts-per-billion levels and will be effective in cleaning mercury and other trace metals in addition to other impurities.
>
> Coal gasification may offer a further environmental advantage in addressing concerns over the atmospheric buildup of greenhouse gases, such as carbon dioxide. If oxygen is used in a coal gasifier instead of air, carbon dioxide is emitted as a concentrated gas stream in syngas at high pressure. In this form, it can be captured and sequestered more easily and at lower costs. By

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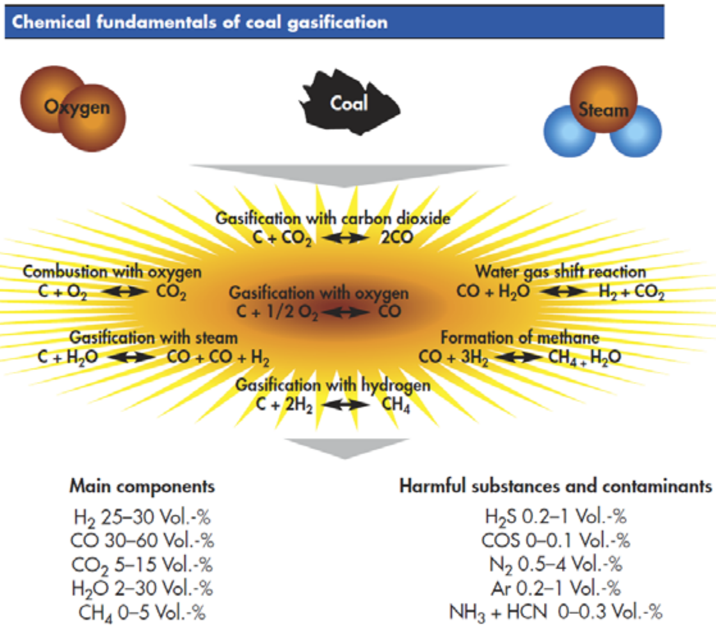
121 http://energy.gov/fe/how-gas-turbine-power-plants-work slight MCM modifications
122 http://energy.gov/fe/science-innovation/clean-coal-research/hydrogen-turbines
123 http://energy.gov/fe/science-innovation/clean-coal-research/hydrogen-coal
124 http://energy.gov/fe/science-innovation/clean-coal-research/gasification
125 ibid.

contrast, when coal burns or is reacted in air, 79 percent of which is nitrogen, the resulting carbon dioxide is diluted and more costly to separate."

The following illustrates some of the reaction intermediates and the final gases produced in coal gasification:[126]




In order to use coal in a combined gas and steam turbine process with between 50 and 60% efficiency, it is necessary to convert the solid fuel coal into a combustible gas. This process step is called gasification. Here, unlike in combustion, a fuel is not completely converted to $CO_2$ and $H_2O$, but mainly to CO and $H_2$. Both solid and liquid fuels are suitable as raw materials for gasification. In the gasification reactor, the fuel reacts with a gasification agent which contains oxygen, usually an oxygen-steam mixture, pure oxygen, or air. The heating necessary for gasification occurs in the reactor itself. This method of operation is called autothermic gasification. Allothermic gasification, in which the heat is fed into the reactor from outside, is not implemented on a large scale.

Gas produced by gasification is sometimes called *syngas*.

> "Syngas, or synthesis gas, is a fuel gas mixture consisting primarily of hydrogen, carbon monoxide, and very often some carbon dioxide. The name comes from its use as intermediates in creating synthetic natural gas (SNG) and for producing ammonia or methanol. Syngas is usually a product of gasification and the main application is electricity generation… Syngas can be produced from many sources, including natural gas, coal, biomass, or virtually any hydrocarbon feedstock, by reaction with steam or oxygen. Syngas is a crucial intermediate resource for production of hydrogen, ammonia, methanol, and synthetic hydrocarbon fuels. Syngas is also used as an intermediate in producing synthetic petroleum for use as a fuel or lubricant… Syngas is combustible and often used as a fuel of internal combustion engines. It has less than half the energy density of natural gas."[127]

### Internal Combustion Reciprocating Engine Turbines

These do not include gas turbines. They are usually fueled by diesel oil, heavy oil, natural gas, and landfill gas,[128] and play an ancillary role.

### Combined Cycle Plants

"Combined cycle plants have both a gas turbine fired by natural gas, and a steam boiler and steam turbine which use the hot exhaust gas from the gas turbine to produce electricity. This greatly increases the overall

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[126] http://www.bine.info/fileadmin/content/Publikationen/Englische_Infos/projekt_0906_engl_internetx.pdf image slightly modified by MCM
[127] https://en.wikipedia.org/wiki/Syngas
[128] https://en.wikipedia.org/wiki/Power_station

efficiency of the plant, and many new baseload power plants are combined cycle plants fired by natural gas."[129]

## Efficiency of Thermal Power Plants

See also discussion of thermodynamic thermal efficiency here.[130]

**Energy Content of Fuels (Thermal Energy Constants)**:

The thermal energy contained in fossil fuels may be expressed as Thermal Energy Constants, defined as the amount of thermal energy produced (in BTUs) per kg of burned fuel.
1 BTU ≈ 1.055 kJ ≈ 1 kJ (exact definitions of the BTU vary).
Typical TEC values (from the textbook *EEAI3*) are: Petroleum liquid 45,000 BTUs/kg, Natural Gas 48,000, Coal 27,000 and Dry Wood 19,000. Much of the heat energy generated by burning is lost as waste heat at the cooling tower, so overall system efficiencies are low, often well below 50%.[131]

**Heat Rate**:

The EIA (U.S. Energy Information Administration) gives the following discussion regarding the efficiency of different types of power plants, expressed as heat rates:

> "One measure of the efficiency of a generator or power plant that converts a fuel into heat and into electricity is the *heat rate*. The heat rate is the amount of energy used by an electrical generator or power plant to generate one kilowatthour (kWh) of electricity. The U.S. Energy Information Administration (EIA) expresses heat rates in British thermal units (Btu) per net kWh generated. Net generation is the amount of electricity a power plant (or generator) supplies to the power transmission line connected to the power plant. Net generation accounts for all the electricity that the plant itself consumes to operate the generator(s) and other equipment, such as fuel feeding systems, boiler water pumps, cooling equipment, and pollution control devices... To express the efficiency of a generator or power plant as a percentage, divide the equivalent Btu content of a kWh of electricity (which is 3,412 Btu) by the heat rate. For example, if the heat rate is 10,500 Btu, the efficiency is 33%. If the heat rate is 7,500 Btu, the efficiency is 45%... EIA only publishes heat rates for fossil fuel-fired generators and nuclear power plants... There is a discussion of the method that EIA uses to estimate the amount of energy consumed to generate electricity with renewable energy sources in Alternatives for Estimating Energy Consumption, which includes a table with estimates for the conversion efficiencies of noncombustible renewable energy sources (geothermal, hydro, solar, and wind energy)."[132]

[129] ibid.
[130] http://www.mcgoodwin.net/pages/thermodynamics.pdf
[131] *EEAI3* p. 71-72
[132] http://www.eia.gov/tools/faqs/faq.cfm?id=107&t=3

The EIA provides the following heat values by year for the stated energy sources (lower values mean greater efficiency):[133]

**Table 8.1. Average Operating Heat Rate for Selected Energy Sources, 2003 through 2013 (Btu per Kilowatthour)**

| Year | Coal | Petroleum | Natural Gas | Nuclear |
|---|---|---|---|---|
| 2003 | 10297 | 10610 | 9207 | 10422 |
| 2004 | 10331 | 10571 | 8647 | 10428 |
| 2005 | 10373 | 10631 | 8551 | 10436 |
| 2006 | 10351 | 10809 | 8471 | 10435 |
| 2007 | 10375 | 10794 | 8403 | 10489 |
| 2008 | 10378 | 11015 | 8305 | 10452 |
| 2009 | 10414 | 10923 | 8160 | 10459 |
| 2010 | 10415 | 10984 | 8185 | 10452 |
| 2011 | 10444 | 10829 | 8152 | 10464 |
| 2012 | 10498 | 10991 | 8039 | 10479 |
| 2013 | 10459 | 10713 | 7948 | 10449 |

> "Coal includes anthracite, bituminous, subbituminous and lignite coal. Waste coal and synthetic coal are included starting in 2002... Petroleum includes distillate fuel oil (all diesel and No. 1 and No. 2 fuel oils), residual fuel oil (No. 5 and No. 6 fuel oils and bunker C fuel oil, jet fuel, kerosene, petroleum coke, and waste oil. Included in the calculation for coal, petroleum, and natural gas average operating heat rate are electric power plants in the utility and independent power producer sectors. Combined heat and power plants, and all plants in the commercial and industrial sectors are excluded from the calculations. The nuclear average heat rate is the weighted average tested heat rate for nuclear units..."

As stated in the discussion above, the efficiency of these fuels in the most recent year (2013) compared to the equivalent Btu content of a kWh of electricity is readily calculated to be 33% for coal, 32% for petroleum, 43% for natural gas, and 33% for nuclear energy, much lower than typical values for hydroelectric power.

Clearly, heat values have not improved much in the past 10 years, with the exception that natural gas heat values have improved by 14%.

Looking at 2013 (the most recent EIA data), the heat values for various prime movers are shown as follows:[134]

[133] http://www.eia.gov/electricity/annual/html/epa_08_01.html including quote that follows immediately.
[134] http://www.eia.gov/electricity/annual/html/epa_08_02.html data selected and edited by MCM

## Table 8.2. Average Tested Heat Rates by Prime Mover and Energy Source, 2007 - 2013

**(Btu per Kilowatthour)**

| Prime Mover | Coal | Petroluem | Natural Gas | Nuclear |
|---|---|---|---|---|
| **2013** | | | | |
| Steam Generator | 10,089 | 10,334 | 10,354 | 10,449 |
| Gas Turbine | -- | 13,555 | 11,371 | -- |
| Internal Combustion | -- | 10,401 | 9,573 | -- |
| Combined Cycle | W | 9,937 | 7,667 | -- |

Notes: W = Withheld to avoid disclosure of individual company data.

Heat rate is reported at full load conditions for electric utilities and independent power producers.
The average heat rates above are weighted by Net Summer Capacity.
Coal Combined Cycle represents integrated gasification units.

We may conclude for 2013 and comparing to the efficiency of the steam generator (about 34% efficiency for all energy sources):

• the *gas turbine* burning petroleum is especially inefficient (only 25% efficiency), but the inefficiency is less pronounced when burning natural gas (30%). But see above about how gas turbines should have high efficiency.

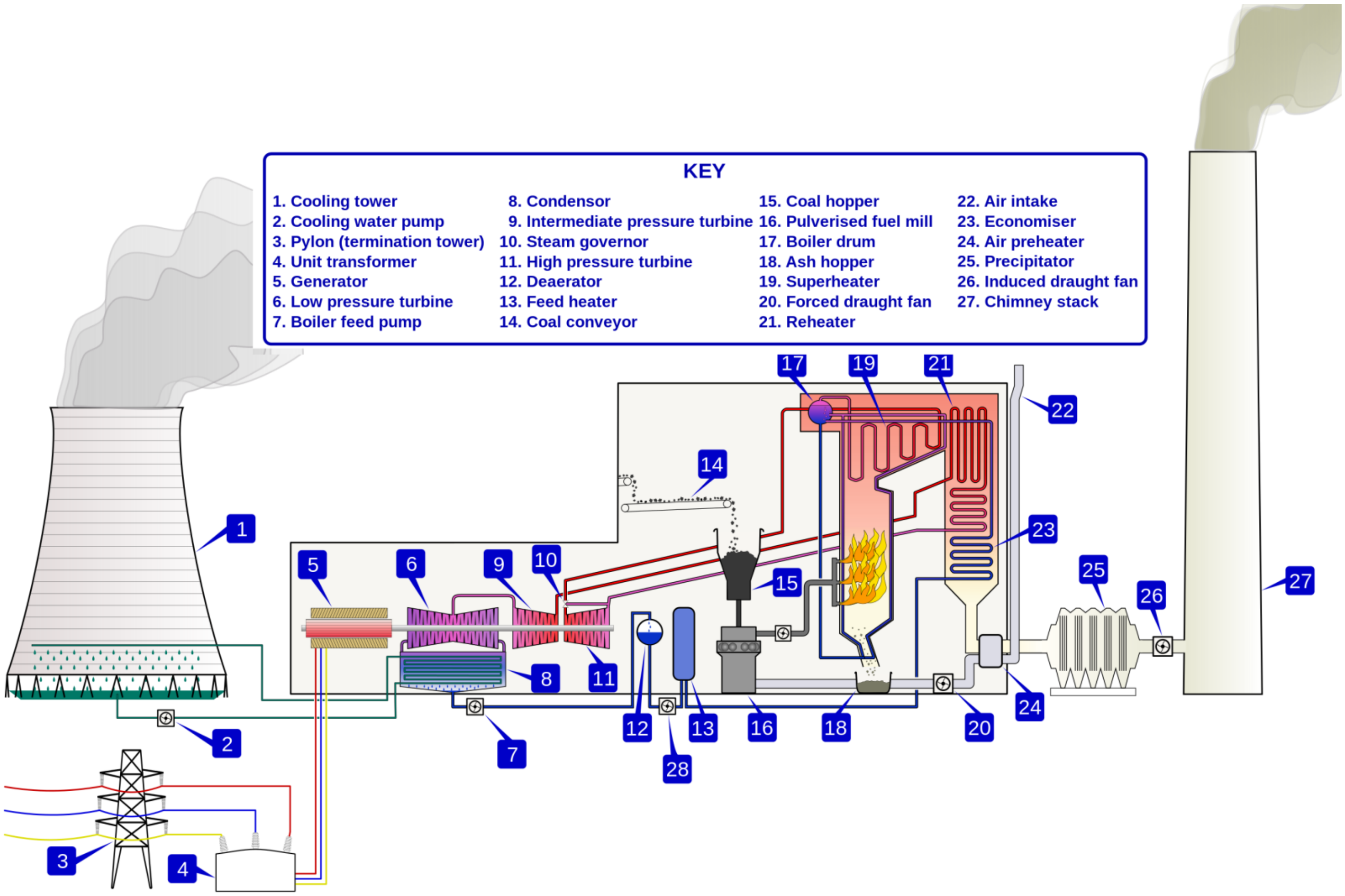
• the *internal combustion* process has about the same efficiency as the steam generator (with 33%), somewhat better with natural gas (36%), and

• that the *combined cycle* when burning petroleum has about the same efficiency as the steam generator (with 34%), but is significantly better when burning natural gas (efficiency 45%).

The last item is explained as follows: "In electric power generation a combined cycle is an assembly of heat engines that work in tandem from the same source of heat, converting it into mechanical energy, which in turn usually drives electrical generators. The principle is that after completing its cycle (in the first engine), the working fluid of the first heat engine is still low enough in its entropy that a second subsequent heat engine may extract energy from the waste heat (energy) of the working fluid of the first engine… In stationary power plants, a widely used combination is a gas turbine (operating by the Brayton cycle) burning natural gas or synthesis gas from coal, whose hot exhaust powers a steam power plant (operating by the Rankine cycle). This is called a Combined Cycle Gas Turbine (CCGT) plant, and can achieve a best-of-class real [HHV=Higher Heating Value] thermal efficiency of around 54% in base-load operation, in contrast to a single cycle steam power plant which is limited to efficiencies of around 35-42%."[135]

A coal fired thermal power plant is illustrated to follow:[136]

[135] https://en.wikipedia.org/wiki/Combined_cycle
[136] https://en.wikipedia.org/wiki/Thermal_power_station
images by https://en.wikipedia.org/wiki/User:BillC , modified minimally by MCM: https://commons.wikimedia.org/wiki/File:PowerStation3.svg and https://en.wikipedia.org/wiki/File:PowerStation2.svg




The person contributing this image offers the following additional key and explanation [minor alterations by MCM]:

> “1. [Hyperboloid wet probably natural draft] cooling tower. 2. Cooling water pump. 3. Transmission line (3-phase). 4. Unit transformer (3-phase). 5. Electric generator (3-phase). 6. Low pressure turbine. 7. Condensate extraction [boiler feed] pump. 8. Condenser. 9. Intermediate pressure turbine. 10. Steam governor valve. 11. High pressure turbine. 12. Deaerator. 13. Feed heater. 14. Coal conveyor. 15. Coal hopper. 16. Pulverised fuel mill. 17. Boiler drum. 18. Ash hopper. 19. Superheater. 20. Forced draught fan. 21. Reheater. 22. Air intake. 23. Economiser. 24. Air preheater. 25. Precipitator. 26. Induced draught fan. 27. Chimney stack. [28. Feed pump between deaerator and feed heater, located differently on image with key].
>
> Coal is conveyed (14) from an external stack and ground to a very fine powder by large metal spheres in the pulverised fuel mill (16). There it is mixed with preheated air (24) driven by the forced draught fan (20). The hot air-fuel mixture is forced at high pressure into the boiler where it rapidly ignites. Water of a high purity flows vertically up the tube-lined walls of the boiler, where it turns into steam, and is passed to the boiler drum, where steam is separated from any remaining water. The steam passes through a manifold in the roof of the drum into the pendant superheater (19) where its temperature and pressure increase rapidly to around 200 bar and 570°C, sufficient to make the tube walls glow a dull red. The steam is piped to the high pressure turbine (11), the first of a three-stage turbine process. A steam governor valve (10) allows for both manual control of the turbine and automatic set-point following. The steam is exhausted from the high pressure turbine, and reduced in both pressure and temperature, is returned to the boiler reheater (21). The reheated steam is then passed to the intermediate pressure turbine (9), and from there passed directly to the low pressure turbine set (6). The exiting steam, now a little above its boiling point, is brought into thermal contact with cold water (pumped in from the cooling tower) in the condenser (8), where it condenses rapidly back into water, creating near vacuum-like conditions inside the condenser chest. The condensed

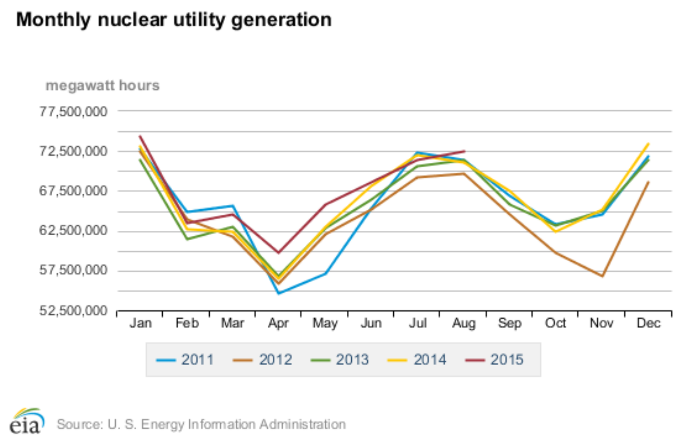
water is then passed by a condensate pump (7) to a deaerator (12), then pumped by feedwater pump (28) and pre-warmed, first in a feed heater (13) powered by steam drawn from the high pressure set, and then in the economiser (23), before being returned to the boiler drum. The cooling water from the condenser is sprayed inside a cooling tower (1), creating a highly visible plume of water vapor, before being pumped back to the condenser (8) in cooling water cycle.

The three turbine sets are sometimes coupled on the same shaft as the three-phase electrical generator (5) which generates an intermediate level voltage (typically 20-25 kV). This is stepped up by the unit transformer (4) to a voltage more suitable for transmission (typically 250-500 kV) and is sent out onto the three-phase transmission system (3).

Exhaust gas from the boiler is drawn by the induced draft fan (26) through an electrostatic precipitator (25) and is then vented through the chimney stack (27)."

## Nuclear Power Plants

These are discussed in the textbook, chapter 4, p. 75-86. This topic warrants much more attention than I have yet been able to give to it, particularly the environmental issues; waste disposal; risks of coolant loss with overheating, hydrolysis at 1200°C, and fuel rod melting at 2400°C; and potential for adverse use by terrorists or other aggressors. Chapter 5 deals in part with nuclear waste disposal and environmental contamination.[137]




Such plants depend on heat generated by nuclear fission (fusion is not yet a viable option), which is used to turn water into steam to drive steam turbines.

Worldwide, there are about 400 commercial nuclear power plants (generating a total[138] in 2014 of 2,410 TWh). Nuclear provides 10.6% of global domestic electrical energy generation (2013 data from IEA)[139] or 19.5% in 2014.[140]

There are 99 operational power reactors in 61 commercial power plants in the US.[141] Typical generating capacity per US reactor in 2014 ranges from 5 to 12 TWh (i.e., billions of kWh),[142] with an average in 2010 of 7.8 TWh per reactor (i.e., 7,759,000 MWh).[143]

U.S. electricity generated from nuclear energy in 2014 totaled 797 billion kWh (797 TWh), which comprised 19.5% of total US electricity generation (4,092 TWh).[144]

In Washington state as of 2013, the boiling water reactor in the Columbia Generating Station 2 near Richland annually generates 8.5 billion kWh (8.5 TWh), comprising 7.5% of Washington electricity generation. Hydroelectric provided 68.8%, natural gas 10%, coal 5.9%, and renewable and other 7.8%.[145] There has been little growth in recent years in nuclear generation of electricity in the US (see graph).[146]

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137 *EEAI3* p. 97
138 http://www.world-nuclear.org/info/Facts-and-Figures/Nuclear-generation-by-country/
139 https://www.iea.org/publications/freepublications/publication/key-world-energy-statistics-2015.html
140 http://www.world-nuclear.org/info/facts-and-figures/nuclear-generation-by-country/
141 *EEAI3* p. 75 and http://www.eia.gov/tools/faqs/faq.cfm?id=207&t=3
142 http://www.eia.gov/nuclear/reactors/stats_table1.html
143 computed by MCM from http://www.eia.gov/nuclear/reactors/stats_table1.xls
144 http://www.nei.org/Knowledge-Center/Nuclear-Statistics/US-Nuclear-Power-Plants and http://www.world-nuclear.org/info/country-profiles/countries-t-z/usa--nuclear-power/
145 http://www.nei.org/CorporateSite/media/filefolder/Backgrounders/Fact-Sheets/Washington-State-Fact-Sheet-2014_1.pdf
146 http://www.eia.gov/nuclear/generation/index.html

Reactors are typically boiling water reactors BWR[147] (35 in the US) or pressurized water reactors PWR[148] (65 in the US). In BWR, water that moves through the reactor core is allowed to boil into steam that is used to turn the turbine directly. In contrast, PWRs keep water under pressure so that it heats, but does not boil. Water from the reactor [radioactive] and the water in the steam generator that is turned into steam never mix..

## Fuel

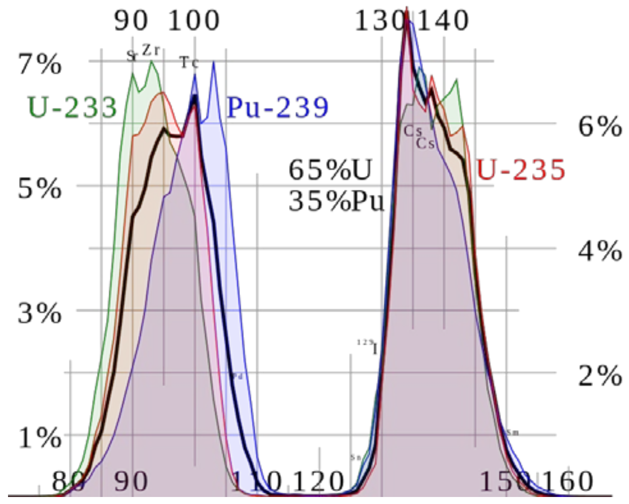
The primary fuels used in power reactors are the *fissile* isotopes of uranium: $^{235}$U enriched to 3 to 5%, with the balance of uranium as the essentially non-fissile isotopes $^{238}$U and $^{234}$U. Some reactors can *breed* $^{239}$Pu from fertile isotopes such as $^{238}$U and $^{232}$Th. A fissile (fissionable) isotope is "capable of sustaining a nuclear fission chain reaction... with neutrons... The predominant neutron energy may be typified by either slow neutrons (i.e., a thermal system [as with $^{235}$U]) or fast neutrons [as with $^{238}$U though much less probable]. Fissile material can be used to fuel thermal-neutron reactors, fast-neutron reactors and nuclear explosives."[149]

The fission of $^{235}$U releases energy as follows:[150]

"The number of neutrons and the specific fission products from any fission event are governed by statistical probability... However, conservation laws require the total number of nucleons and the total energy to be conserved. The fission reaction in U-235 produces fission products such as Ba, Kr, Sr, Cs, I and Xe with atomic masses distributed around 95 and 135. Examples may be given of typical reaction products, such as:

U-235 + n → Ba-144 + Kr-90 + 2n + about 200 MeV
U-235 + n → Ba-141 + Kr-92 + 3n + 170 MeV
U-235 + n → Zr-94 + Te-139 + 3n + 197 MeV

In such an equation, the number of nucleons (protons + neutrons) is conserved, e.g. 235 + 1 = 141 + 92 + 3, but a small loss in atomic mass [the mass deficit] may be shown to be equivalent to the energy released [through $E = mc^2$]. Both the barium and krypton isotopes subsequently decay and form more stable isotopes of neodymium and yttrium, with the emission of several electrons from the nucleus (beta decays). It is the beta decays, with some associated gamma rays, which make the fission products highly radioactive [initially]...



Probability yields from fission (shown as % probability) for $^{233}$U, $^{235}$U, $^{239}$Pu, and a U-Pu mixture. Horizontal axis is atomic mass number (i.e., nucleon number A = Z + N) $^{235}$U peaks on left at A ≈ 95 and on the right at A≈134

The total binding energy released in fission of an atomic nucleus varies with the precise break up, but averages about 200 MeV for U-235 or 3.2 x $10^{-11}$ joule. [These are total energy release figures]... That from U-233 is about the same, and that from Pu-239 is about 210 MeV per fission. (This contrasts with 4 eV or 6.5 x 10-19 J per atom of carbon burned in fossil fuels.)"

A clearer analysis to follow of average fission energy released by $^{235}$U fission explicitly deals with the energy carried away by neutrinos from beta decay. As the table further below shows, "the fission of one atom of U-235 generates [an average of ] 202.5 MeV = 3.24 × $10^{-11}$ J, which translates to 19.54 TJ/mol, or 83.14 TJ/kg."[151]

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147 http://www.nrc.gov/reading-rm/basic-ref/students/animated-pwr.html animation
148 http://www.nrc.gov/reading-rm/basic-ref/students/animated-bwr.html animation
149 https://en.wikipedia.org/wiki/Fissile_material
150 http://www.world-nuclear.org/info/Nuclear-Fuel-Cycle/Introduction/Physics-of-Nuclear-Energy/ quoted & paraphrased MCM
151 https://en.wikipedia.org/wiki/Uranium-235

To compare coal to nuclear: The thermal energy content of 1 kg of coal is 29.3 MJ or $2.93 \times 10^7$ J.[152] From above, the thermal energy content of $^{235}$U is 83.14 TJ/kg = $8.314 \times 10^{13}$ J. On a per weight basis and neglecting various inefficiencies, 1 kg of $^{235}$U is equivalent to 2,837,543 kg of coal, so the ratio is $2.84 \times 10^6$.

The World Nuclear Association WNA estimates that to produce 1 GW of electricity over a year requires about 27 tonnes of fresh enriched uranium fuel each year. This is needed to keep the gradually depleting 75 tonnes of low enriched uranium in the reactor adequately replenished. The WNA also states,[153]

> "An issue in operating reactors and hence specifying the fuel for them is fuel burn-up. This is measured in gigawatt-days per tonne [GWd/t of enriched fuel] and its potential is proportional to the level of enrichment. Hitherto a limiting factor has been the physical robustness of fuel assemblies, and hence burn-up levels of about 40 GWd/t have required only around 4% enrichment. But with better equipment and fuel assemblies, 55 GWd/t is possible (with 5% enrichment), and 70 GWd/t is in sight, though this would require 6% enrichment. The benefit of this is that operation cycles can be longer – around 24 months – and the number of fuel assemblies discharged as used fuel can be reduced by one third. Associated fuel cycle cost is expected to be reduced by about 20%.

The following table[154] clarifies that the energy of the [approximately 6 electron-type[155]] anti-neutrinos (8.8 MeV, arising from beta-decays) is carried away and does not contribute to reactor heat generation. A confusingly similar amount of energy, 8.8 MeV for $^{235}$U, is added to the reactor heat when certain of the neutrons that were released in a fission reaction are finally captured by nuclei but do not themselves lead to new fission. Thus, although an average of 211.3 MeV of energy in multiple forms including anti-neutrinos are released per fission, only 202.5 MeV are available to add to reactor heat and therefore electrical generation.

| Source | Average energy released [MeV] |
|---|---|
| Instantaneously released energy | |
| Kinetic energy of fission fragments | 169.1 |
| Kinetic energy of prompt neutrons | 4.8 |
| Energy carried by prompt γ-rays | 7.0 |
| Energy from decaying fission products | |
| Energy of $\beta^-$-particles | 6.5 |
| Energy of delayed γ-rays | 6.3 |
| Energy released when those prompt neutrons which don't (re)produce fission are captured | 8.8 |
| Total energy converted into heat in an operating thermal nuclear reactor | 202.5 |
| Energy of anti-neutrinos | 8.8 |
| Sum | 211.3 |

Reactor neutrons are categorized by energy (though definitions seem to vary widely):[156]

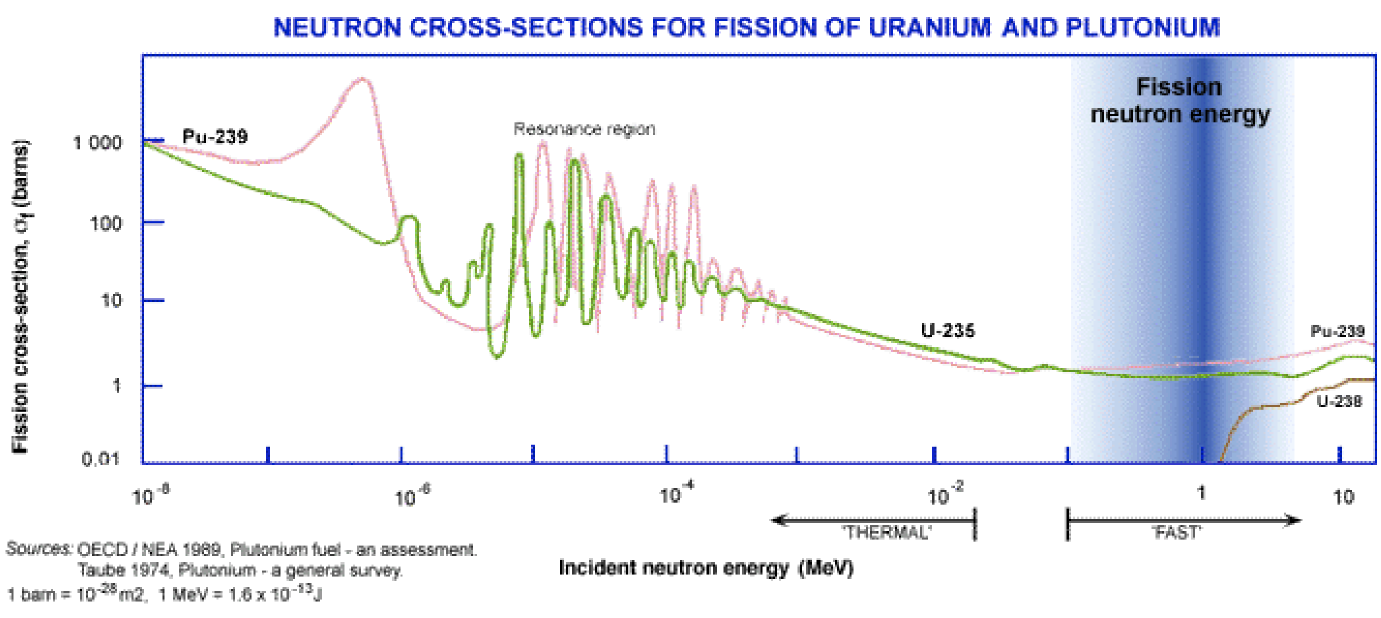
- *A Thermal neutron* is a free neutron with a kinetic energy of about 0.025 eV (about $4.0 \times 10^{-21}$ J or 2.4 MJ/kg, hence a speed of 2.2 km/s), which is the energy corresponding to the most probable velocity at a temperature of 290 K…
- *An Epithermal neutron* has 0.025 eV < KE < 1 eV (however, definitions vary)

152 https://www.euronuclear.org/info/encyclopedia/coalequivalent.htm
153 http://www.world-nuclear.org/info/Nuclear-Fuel-Cycle/Introduction/Nuclear-Fuel-Cycle-Overview/
154 https://en.wikipedia.org/wiki/Uranium-235
155 http://t2k-experiment.org/neutrinos/sources-and-experiments/
156 https://en.wikipedia.org/wiki/Neutron_temperature , also http://www.nuclearglossary.com/suites/nuclearglossary_neutrons.html

- A *Fast neutron* is a free neutron with a kinetic energy level close to 1 MeV (100 TJ/kg), hence a speed of 14,000 km/s or higher… [Some authors use 100 keV or even 10 keV as the lower cutoff value.[157]] Nuclear fission produces neutrons with a mean energy of 2 MeV (200 TJ/kg, i.e. 20,000 km/s)… However the range of neutrons from fission follows a Maxwell–Boltzmann distribution from 0 to about 14 MeV… and the mode of the energy is only 0.75 MeV, meaning that fewer than half of fission neutrons qualify as "fast" even by the 1 MeV criterion.

The following quoted text and log-log graph describe the probability of fission interactions for $^{235}$U and $^{239}$Pu. The "fission neutron energy" presumably refers to the range of energy of neutrons given off during fission:[158] (Note: a useful table of all credible isotopes, including radioisotopes, is given here.[159] This facilitating assessment of odd vs. even numbers of neutrons.)

> "Fission may take place in any of the heavy nuclei after capture of a neutron. However, low-energy (slow, or thermal) neutrons are able to cause fission only in those isotopes of uranium and plutonium whose nuclei contain odd numbers of neutrons (e.g. U-233, U-235, and Pu-239). Thermal fission may also occur in some other transuranic elements whose nuclei contain odd numbers of neutrons. For nuclei containing an even number of neutrons, fission can only occur if the incident neutrons have energy above about one million electron volts (MeV) ["fast" neutrons]. (Newly-created fission neutrons are in this category and move at about 7% of the speed of light, while moderated neutrons move a lot slower, at about eight times the speed of sound.)
>
> The probability that fission or any another neutron-induced reaction will occur is described by the neutron cross-section for that reaction. The cross-section may be imagined as an area surrounding the target nucleus and within which the incoming neutron must pass if the reaction is to take place. The fission and other cross sections increase greatly as the neutron velocity reduces from around 20,000 km/s to 2 km/s, making the likelihood of some interaction greater. In nuclei with an odd-number of neutrons, such as U-235, the fission cross-section becomes very large at the thermal energies of slow neutrons."

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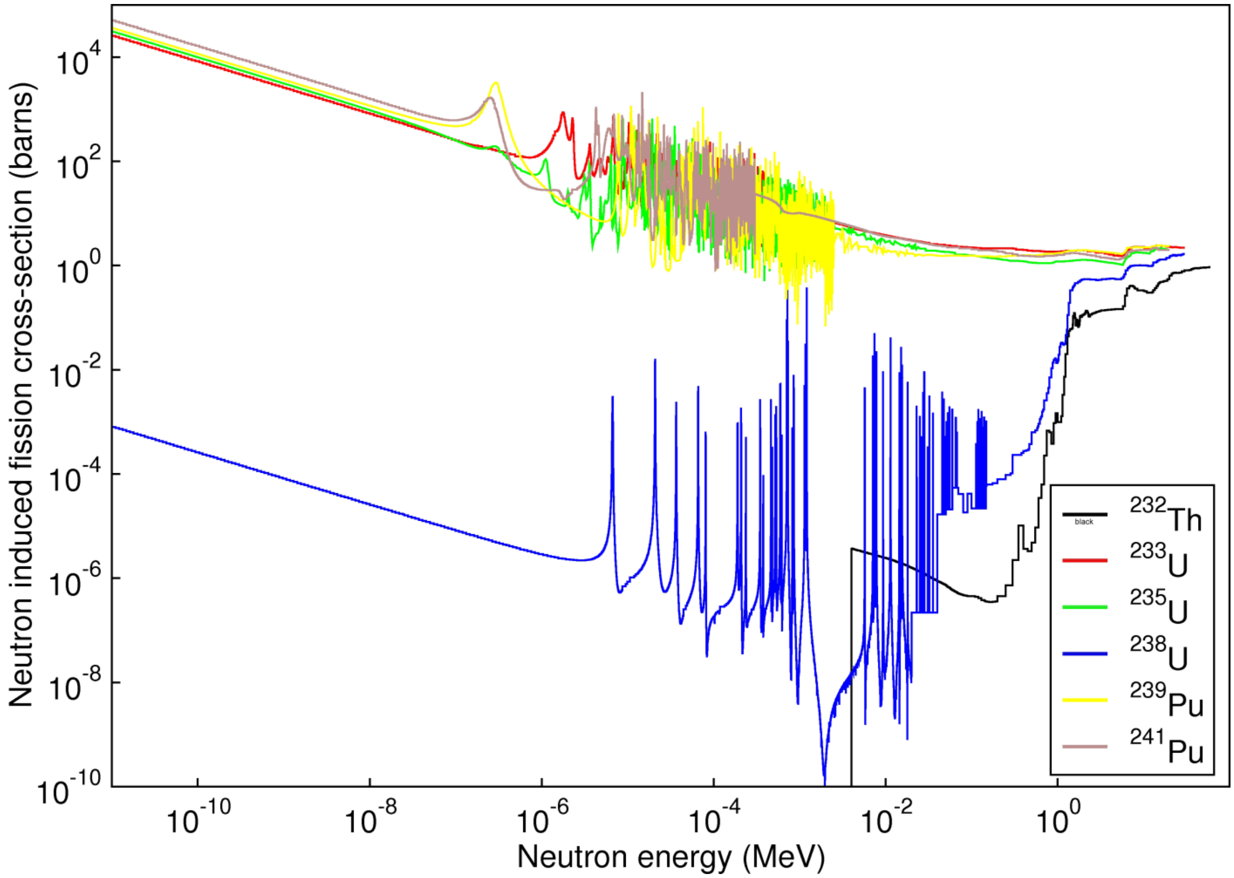
[157] http://ocw.mit.edu/courses/nuclear-engineering/22-55j-principles-of-radiation-interactions-fall-2004/lecture-notes/ener_depo_neutro.pdf

[158] http://www.world-nuclear.org/info/Nuclear-Fuel-Cycle/Introduction/Physics-of-Nuclear-Energy/

[159] https://en.wikipedia.org/wiki/Table_of_nuclides_%28complete%29

The following graph,[160] also in log-log format, depicts the fission cross sections for 6 isotopes: the 2 lowermost curves (with lowest probability, colors black and blue) are for $^{238}U$ and $^{232}Th$, while the upper four, all rather similar, are for $^{233}U$, $^{235}U$, $^{239}Pu$, and $^{241}Pu$.




A summary of fission cross sections for many isotopes may be found here.[161]

There is ongoing modest interest in developing reactors that can use the *fertile* (non-fissile) radioisotope of Thorium $^{232}Th$, which is also a primary fuel found in nature. It is more abundant than uranium, in a reactor it breeds fissile $^{233}U$, and is well suited for molten salt reactors, but the issues appear to be complex and I have not delved much into this subtopic.[162]

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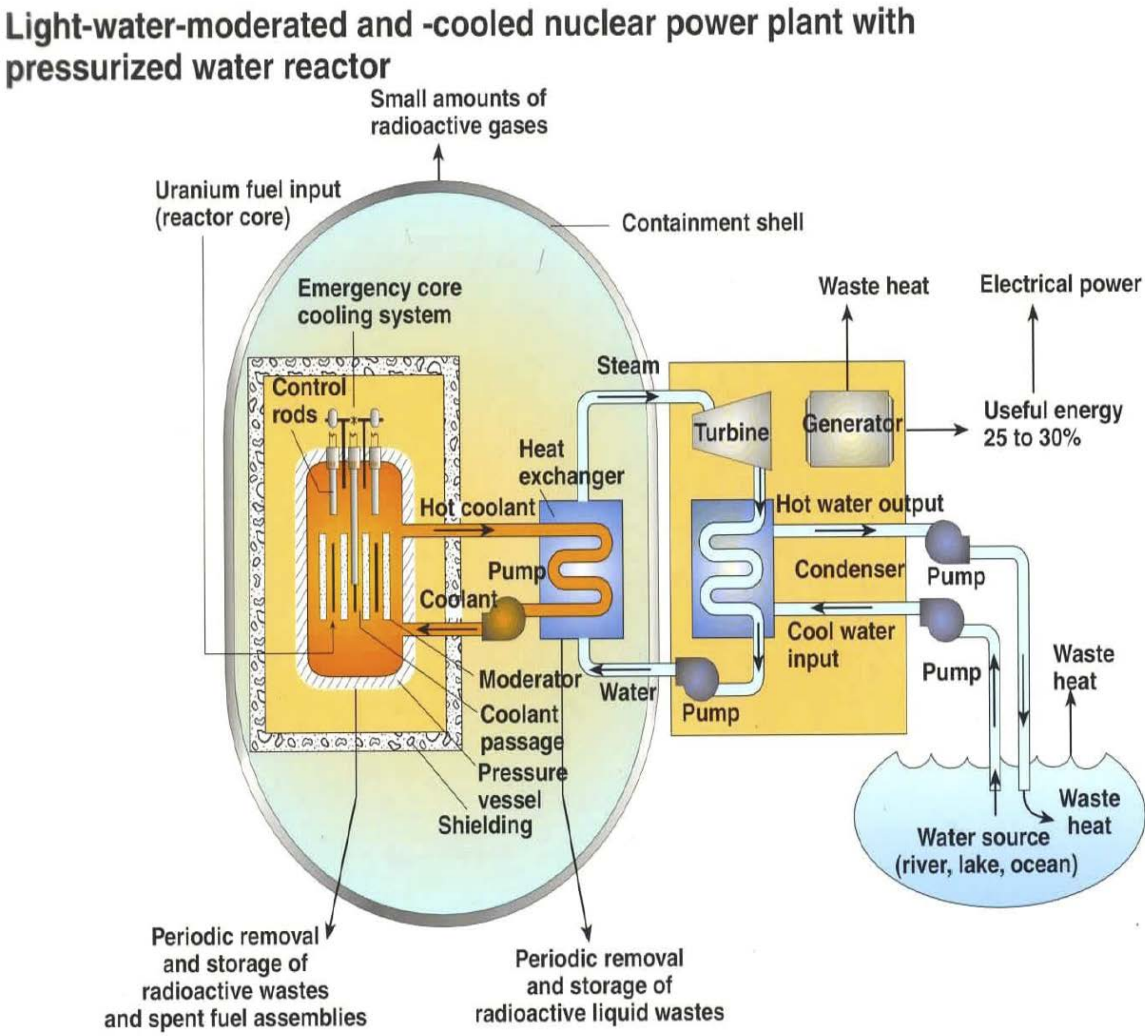
[160] http://thorea.wikia.com/wiki/Thermal,_Epithermal_and_Fast_Neutron_Spectra
[161] http://nuclearweaponarchive.org/Nwfaq/Nfaq12.html
[162] http://www.world-nuclear.org/info/current-and-future-generation/thorium/

### Nuclear Power Plant Design

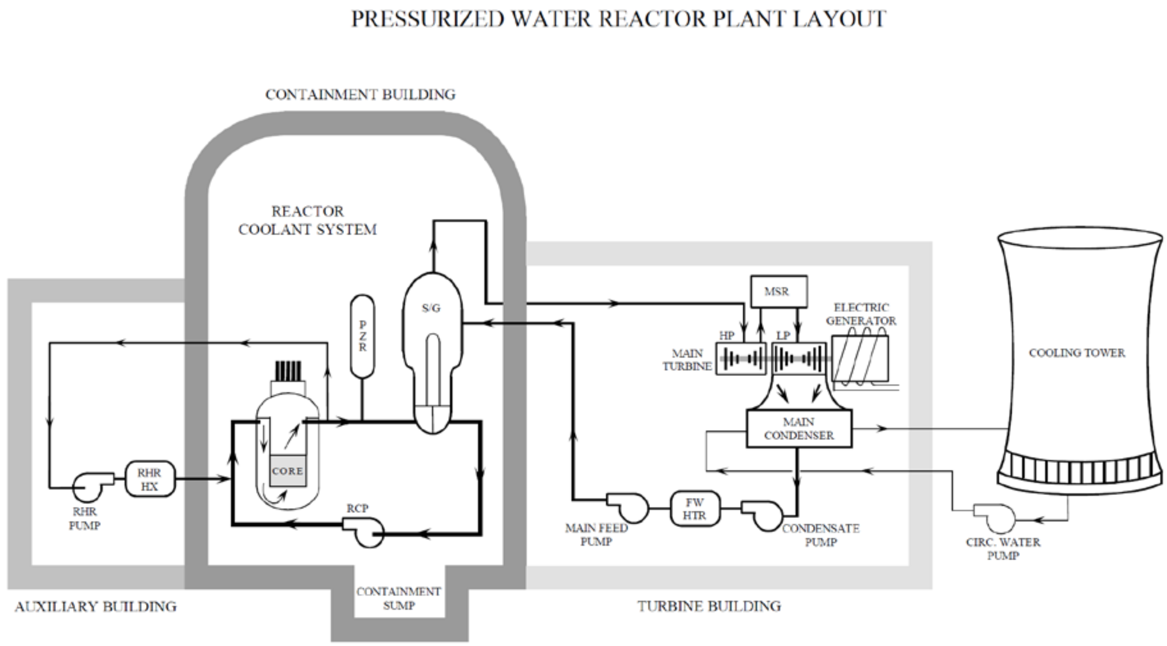
The following diagram depicts a representative Pressurized Water Reactor (PWR) plant:[163]




Not shown is the cooling tower to handle the waste heat.

[163] http://josiah.berkeley.edu/2008Spring/ER102/Handouts/ER102_nuclear_power_handout.pdf

Another diagram of a Pressurized Water Reactor (PWR) plant follows:[164]




Possibly unfamiliar acronyms :[165] RHR = residual heat removal system;
RHR HX = RHR Heat Exchanger; PZR = Pressurizer; SG = steam generator;
RCP = reactor coolant pump; HP = high pressure [turbine]; LP = low pressure [turbine];
MSR = moisture separator reheater; FW HTR = feedwater heater

I have not discussed the role (for conventional reactors) of

- the moderator (for slowing neutrons, usually water $H_2O$, heavy water $D_2O$, or graphite C),
- control rods (for controlling the rate of the nuclear chain reaction, especially boron-10, silver-107, indium-115, cadmium-113, and Hafnium isotopes),[166]
- the pressure vessels and tubes, and
- the containment structure.

Reactors can be classified by fuel, moderator, coolant, generation, fuel phase (liquid or solid), or use. Other types of reactors that are or someday may be employed in nuclear power plants include:[167]

- Pressurized Heavy Water Reactor (PHWR, CANDU)—in Canada, India, uses $^2H$ deuterium as coolant and moderator
- Advanced Gas-cooled Reactor (AGR & Magnox)—in UK, uses $CO_2$ as coolant and graphite as moderator. The Magnox Gas-cooled Reactor design was supplanted by the AGR.
- Light Water Graphite Reactor (RBMK & EGP)—in Russia, uses water as coolant and graphite as moderator. RBMK = Reaktor Bolshoy Moschnosti Kanalniy = high-power channel reactor. Chernobyl was of this design. EGP are small scaled down versions of the RBMK.
- Fast Neutron Reactor (FBR)[168]—in Russia, uses liquid sodium as coolant and no moderator. More advanced designs under development.
- Liquid-metal fast-breeder reactor (LMFBR)
- Other Advanced Reactors under development: Generation III[169] and Generation IV[170] reactors. The latter include:

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[164] http://www.nrc.gov/reading-rm/basic-ref/students/for-educators/01.pdf
[165] http://www.nrc.gov/reading-rm/doc-collections/nuregs/staff/sr0544/
These NRC compiled abbreviations are "in common use in the nuclear industry and regulatory community."
[166] http://large.stanford.edu/courses/2011/ph241/grayson1/
[167] http://www.world-nuclear.org/info/Nuclear-Fuel-Cycle/Power-Reactors/Nuclear-Power-Reactors/
[168] http://www.world-nuclear.org/info/Current-and-Future-Generation/Fast-Neutron-Reactors/
[169] https://en.wikipedia.org/wiki/Generation_III_reactor

- Gas-cooled fast reactor GFR
- Lead-alloy [cooled] fast reactor LFR
- Molten salt reactor MSR
- Sodium-cooled fast reactor SFR
- Supercritical water reactor SCWR
- Very-high-temperature gas reactor VHTR (including Pebble-bed reactors PBR).[171]
  Hydrogen gas production will be an integral part of the VHTR.

A detailed 2003 color presentation for the Gen IV reactor candidates is given here.[172] A 2014 update to the Gen. IV goals, published by the GenIV International Forum (GIF), is given here.[173]

## Renewable Energy Resources

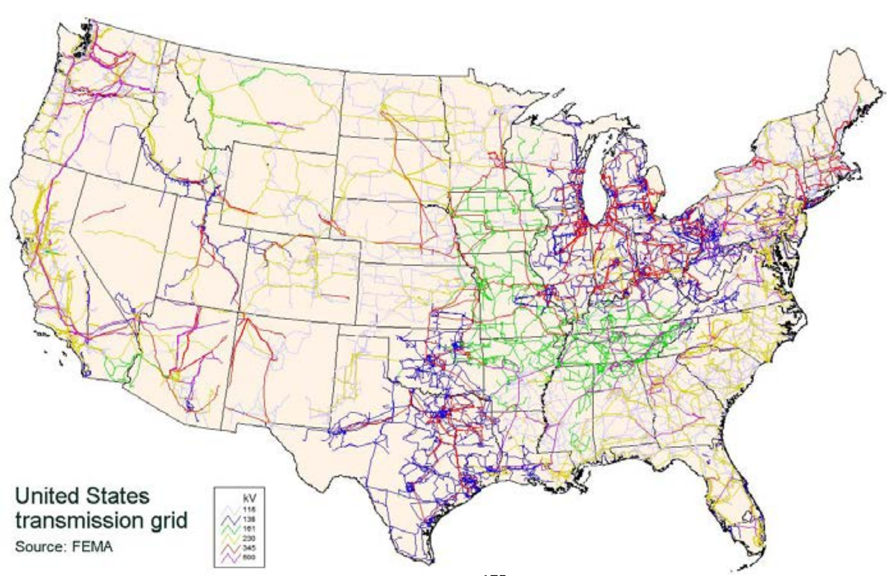
Material covered on Solar, Wind, Geothermal, Biomass, and Hydrokinetic electricity generation derives mostly from chapter 6.[174] Regrettably, I did not find time to do a summary of this important and far-reaching topic.

## Electrical Transmission

Material here derives in part from chapter 2, chapter 7, and chapter 8.

### US Electric Transmission Grid

Most electrical power is transmitted long distances by means of 3 high voltage conductors carrying 3-phase AC voltage. There is usually no neutral conductor, though there may be a ground conductor. In recent years, there has been increasing interest in DC transmission lines for certain situations.



US (lower 48) Power Grid c. 2007[175] , does not show DC lines

Example of one of the few DC interties in the US.[176]

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[170] http://www.world-nuclear.org/info/nuclear-fuel-cycle/power-reactors/generation-iv-nuclear-reactors/
[171] https://en.wikipedia.org/wiki/Generation_IV_reactor and https://en.wikipedia.org/wiki/Nuclear_reactor#Reactor_types
[172] http://www.engr.utk.edu/nuclear/colloquia/slides/Gen%20IV%20U-Tenn%20Presentation.pdf
[173] https://www.gen-4.org/gif/upload/docs/application/pdf/2014-03/gif-tru2014.pdf
[174] *EEAI3* p. 99-212.

## Categories of Transmission Voltage

(per IEC 60038 = International Electrotechnical Commission)[177]

| | | | |
|---|---|---|---|
| Low Voltage | ≤ 1000 V | Medium Voltage | 1,000 to 35,000 V |
| High Voltage | 35 kV to 230 kV | Extra High Voltage | >230 kV |
| Ultra High Voltage | > 800 kV (up to as high as 1150 kV[178]) | | |

## Transmission and Distribution Line Conductors[179]

Conductors are stranded (made of multiple strands for greater flexibility). Aluminum is not quite as good a conductor as copper, but it is much lighter and cheaper and thus is preferred.

**AAC** All Aluminum Conductor. short spans in coastal areas to reduce corrosion.
**AAAC** All Aluminum Alloy Conductor: good strength and corrosion resistance
**ACSR** Aluminum Conductor Steel Reinforced: stronger for longer spans but galv. steel can corrode.
**ACAR** Aluminum Conductor Aluminum-Alloy Reinforced: reduced corrosion, stronger.

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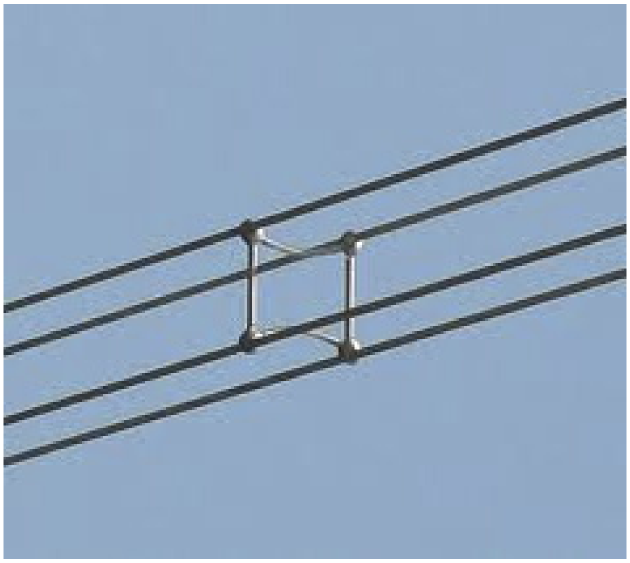
175 https://commons.wikimedia.org/wiki/File:UnitedStatesPowerGrid.jpg fr. FEMA, ?2007 or earlier, does not show HVDC lines such as Pacific DC Intertie, Path 27, etc.
176 https://en.wikipedia.org/wiki/Pacific_DC_Intertie see also https://en.wikipedia.org/wiki/WECC_Intertie_Paths
177 http://www.electrotechnik.net/2011/03/voltage-classification-lv-mv-and-ehv.html
178 https://en.wikipedia.org/wiki/Ekibastuz%E2%80%93Kokshetau_high-voltage_line
179 http://www.southwire.com/support/TransmissionConductoraReviewOfTheDesignandSelectionCriteria.htm



### Bundled High Voltage Conductors

On very high voltage transmission lines with V > 230 kV, multiple *subconductor* cables are bundled together but held apart by (typically) conducting spacers.[180] These multiple connected subconductor cables (consisting of many flexible strands of conducting metal) function like a single conductor and together carry a single phase of the (typically 3-phase) voltage. They are used to reduce electric field strength and energy losses (and noise) due to corona discharge. This is because E (electric field) is roughly a function of V/d where d is the effective overall diameter. The diameter d is effectively increased by the spacing out of the subconductors. These have lower reactance and electric field, as well as improved AC skin effect, but at the cost of greater wind resistance and expense.

### Typical Double Circuit High Voltage Power Line Configuration and Power Transmitted

In the photo above right (of a power line in England),[181] I infer the following likely characteristics:

Terminology about these HV wires, cables, and conductors can be confusing and inconsistent. I will refer to what carries a single phase as a conductor; the components of a bundled conductor are subconductors consisting of multiple strands. A conductor carrying one phase is a *line* or *phase*.

A *ground* / *Earth* conductor is the single conductor/wire at the very top. There are 2 parallel sets of naked 3-phase high voltage lines (i.e., two 3-phase circuits), 6 conductors in all and spaced widely apart. Each 3-phase conductor (*phase* or *line*) consists of 4 bundled subconductors, with each conductor bundle separated from the metal tower by long insulators. It is likely that the 2 sets of 3-phase lines depicted here carry exactly the same voltage and phase because they originate at a common point. (The lines however may be "transposed" along the way to achieve "optimum phasing".)[182]

Circuits are often doubled to reduce inductive reactance (and resistance if applicable) and thereby increase the combined current carrying capacity. In a rough calculation from the textbook,[183] maximum power of a transmission line is given by

$$P_{max} = \frac{3V_0E_f}{X}$$

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[180] http://www.electrotechnik.net/2011/02/bundled-conductors-in-tranmission-lines.html
(image slightly modified MCM);
see also https://en.wikipedia.org/wiki/Overhead_power_line#Bundled_conductors

[181] https://en.wikipedia.org/wiki/Electric_power_transmission

[182] https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/37446/1255-code-practice-optimum-phasing-power-lines.pdf

[183] *EEAI3* p. 448 (in chapter 12)

where $V_0$ = Voltage of the *infinite bus* to which the transmission line connects
$E_f$ = Equivalent rms field voltage
X = total inductive reactance, combining that from the synchronous generator $X_s$ and of the transmission line $X_l$

The resistance is said to be negligible compared to the inductive reactance. Presumably, total inductive reactance X here has factored in any reduction by capacitive reactance.

If $V_0$ = 230 kV, $E_f$ = 210 kV, $X_l$ = 10 Ω, and $X_s$ = 2 Ω, this single 3-phase circuit can carry

$$P_{max} = \frac{3V_0E_f}{X} = \frac{230 \times 210}{(2+10)} = 4.0 \text{ GW}$$

If the transmission line is doubled, the maximum power rises to:

$$P_{max} = \frac{3V_0E_f}{X} = \frac{230 \times 210}{(2+\frac{10\times10}{10+10})} = 6.9 \text{ GW}$$

Doubling the transmission lines in this example produces a 71% increase in $P_{max}$, thus not a full doubling of $P_{max}$.

Transmission line inductive reactance is also reduced (and thus $P_{max}$ raised) by the use of bundling described above. However, inductive reactance is increased by increasing spacing between the phases, a necessity with high voltage lines. More details follow

Note: It is not yet entirely clear to me why $E_f$ can be less than $V_0$ and yet the generator imparts energy to the infinite bus.

## Transmission Line Inductive Reactance

Inductive reactance should be reduced as much as possible. The following elegant resource on mathematical analysis of transmission lines, far beyond my skill level,[184] derives the following relationships and provides the quoted text in this section (which I have attempted to interpret):

### Single Conductor

For a single straight wire of infinite length, the combined internal and external inductance is given by

$$L_{tot} = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{GMR}\right) \text{(H/m)}$$

where $L_{tot}$ = Total inductance in H/m of a single conductor carrying A/C and concentrated at the skin surface
$\mu_0$ = vacuum permeability (magnetic constant) = $4\pi \times 10^{-7}$ H $m^{-1}$
D = distance from conductor center to the sampling point (m)
GMR = Geometric Mean Radius for the conductor (m). "GMR = $e^{-1/4}$ r = 0.7788 r. GMR can be considered as the radius of a fictitious conductor assumed to have no internal flux but with the same inductance as the actual conductor with radius r."

### Two-Wire, Single-Phase Line:

The two wires or conductors A and B are assumed to be parallel, solid, and carrying antiparallel currents. They are separated by on-center distance D > r, and each conductor has true radius r. They are linked at point P by magnetic fluxes $\lambda_{AAP}$, $\lambda_{ABP}$, $\lambda_{BAP}$, and $\lambda_{BBP}$. If the sampling point P is considered to be shifted to infinity, and A and B have identical radii, then total inductance per m for the one-phase system is:

$$L_{1-\text{phase system}} = \frac{\mu_0}{\pi}\ln\left(\frac{D}{GMR}\right) \text{(H/m)}$$

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[184] Manuel Reta-Hernández, Chapter 14 in *Electric Power Generation, Transmission, and Distribution, Third Edition,* 2012, entire book available from CRCNetBase through a university proxy. (An earlier 2006 version without access restriction may be found here.) These resources could provide hours of pleasurable reading.

where $L_{1\text{-phase system}}$ = Total inductance in H/m of the combined conductor system carrying A/C
D = on-center distance between the two conductors (m)
GMR = Geometric Mean Radius for each conductor (m)"

The inductance for this system is twice that of a single conductor.

Here, the GMR applies to each conductor. If the conductor is stranded, $GMR_{stranded}$ is usually available from the manufacturer (otherwise the computation is rather involved).

If the conductors are bundled as 4 stranded subconductors per bundle with spacing d (m) between nearest neighbors, the GMR for the bundle becomes:

$$GMR_{4\text{ bundle connectors}} = 1.09\sqrt[4]{GMR_{stranded}d^3}$$

where d = on-center distance between bundle subconductors
$GMR_{stranded}$ = Geometric Mean Radius for each stranded subconductor (m)"

Here, d is greater than GMR. Inspection of this relationship suggests that $GMR_{4\text{ bundle connectors}} > GMR_{stranded}$. Therefore, the GMR appearing in the L equation is larger, so overall inductance is smaller for bundling.

## Balanced 3-Phase Transmission Line with Phases Arranged Symmetrically

In this idealized example, the three conductors A, B, and C are again assumed to be parallel. They are separated by equal on-center distance D > r. The inductance per m for each phase of the three-phase system is:

$$L_{phase} = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{GMR_{phase}}\right)\ (H/m)$$

where $L_{phase}$= Inductance in H/m of one phase conductor
D = on-center distance between any two phase conductors (m)
$GMR_{phase}$ = Geometric Mean Radius for each phase conductor, bundled or not (m)

This value is the same as for a single conductor. For the three phases, I infer that the total system inductance would be multiplied by 3, so

$$L_{3-\text{phase symmetrical system}} = \frac{3\mu_0}{2\pi}\ln\left(\frac{D}{GMR_{phase}}\right)\ (H/m)$$

Thus, total system inductance for this 3-phase arrangement is only 1.5 times that of the single phase system.

Although it is usually infeasible to maintain a symmetrical arrangements over long distance, "it is possible to assume symmetrical arrangement in the transmission line by *transposing* the phase conductors. In a transposed system, each phase conductor occupies the location of the other two phases for one-third of the total line length…"

# Power Electronics

This section is based in part on chapter 10. I have little experience in this field, can only touch on a few of the many relevant items, and will not attempt to review semiconductor principles in detail. The course Lab 1 dealt with power electronics, but as an auditor I was not to be present while the labs were in progress. However, the equipment was demonstrated to me by the knowledgeable TA and 2nd year MSEE graduate student, John J. Sealy. (Incidentally, John advises me that budding experimentalists may obtain a wide array of electronic components from Digi-Key[185] and SparkFun.[186])

The term *Power Electronics* (PE) has come to mean the application of solid-state [SS] electronics to the control and conversion of electric power. Electrical energy here is expressed in Watts rather than as bytes or informational signals, etc.) Use of Power Electronics, including in high voltage applications, has been rapidly increasing in recent decades.

Energy conversion and switching appear integral to power electronics: One author states,

> "Power Electronics is the art of converting electrical energy from one form to another in an efficient, clean, compact, and robust manner for convenient utilisation… In Power Electronics all devices are operated in the switching mode - either 'FULLY-ON' or 'FULLY-OFF'…
>
> Power Electronics involves the study of

- Power semiconductor devices - their physics, characteristics, drive requirements and their protection for optimum utilisation of their capacities,
- Power converter topologies involving them,
- Control strategies of the converters,
- Digital, analogue and microelectronics involved,
- Capacitive and magnetic energy storage elements,
- Rotating and static electrical devices,
- Quality of waveforms generated,
- Electro Magnetic and Radio Frequency Interference
- Thermal Management"[187]

*Power conversions* include:[188]

- • *AC to DC* (Rectifier, Mains power supply unit (PSU), Switched-mode power supply, etc.)
- *DC to AC* (Inverter)
- *DC to DC* (DC-to-DC converter, Voltage Regulator, Linear Regulator, etc.)
- *AC to AC* (Transformer/autotransformer, Voltage converter, Voltage regulator, Cycloconverter, Variable-frequency transformer

Some of these are demonstrated below.

The precursors to SS power electronics include vacuum tube devices (beginning with vacuum diodes, invented by John Fleming in 1904) and the transistor (1947). The transistor was named by John R. Pierce of Bell Labs, who stated, "[The name is] an abbreviated combination of the words 'transconductance' or 'transfer', and 'varistor'. The device logically belongs in the varistor family, and has the transconductance or transfer impedance of a device having gain, so that this combination is descriptive."[189]

SS PE devices may be divided into three types, on the basis of numbers of semiconducting layers:

Two layer devices such as Diodes
Three layer devices such as Transistors,
Four layer devices such as Thyristors.

There are also hybrid devices: Insulated Gate Bipolar Transistor IGBT and Darlington transistor.

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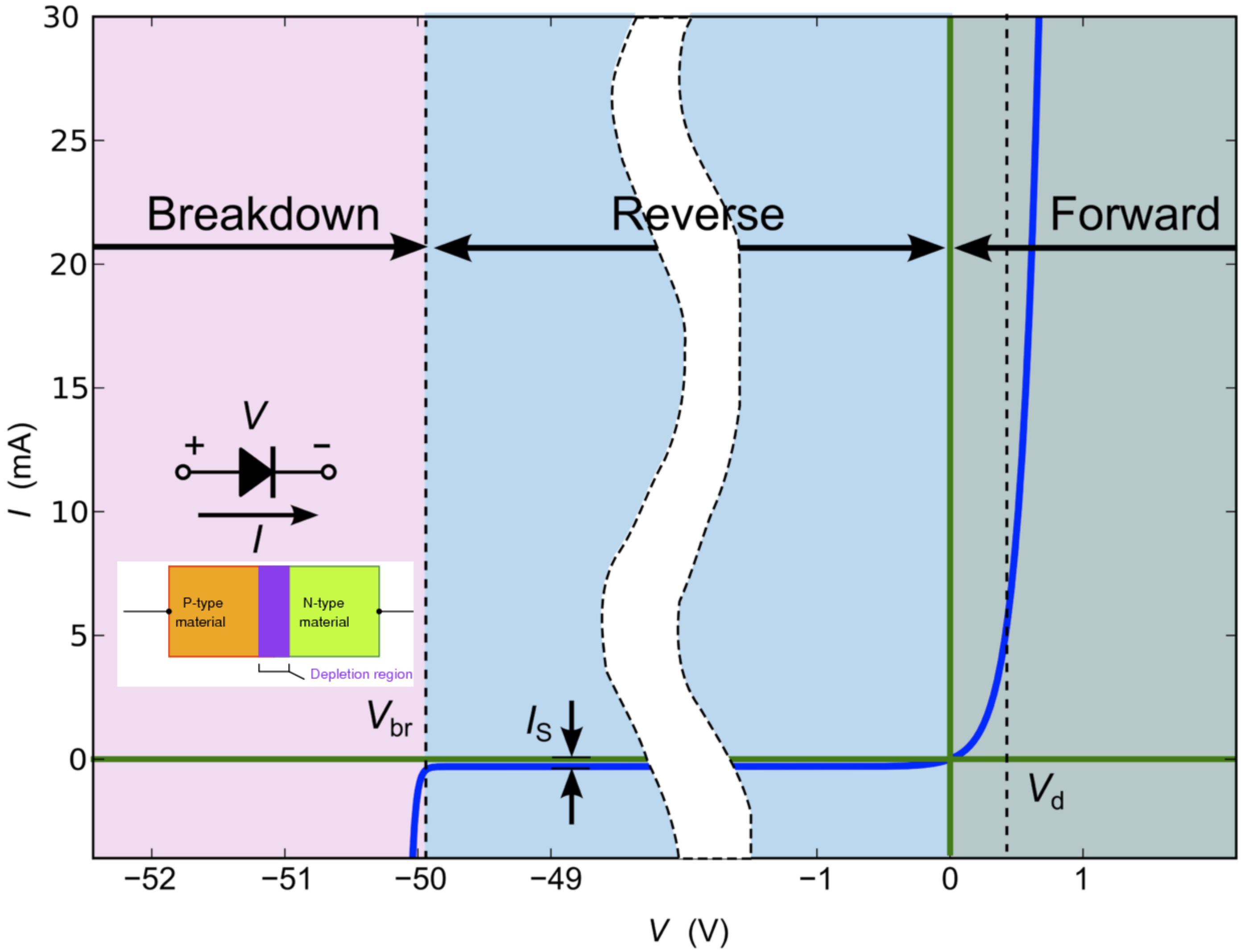
185 http://www.digikey.com/ for new mainstream industrial parts
186 https://www.sparkfun.com/ for new and used parts, hobbyist and industrial
187 Kharagpur. "Power Semiconductor Devices", Module 1, Lesson 1, in http://www.nptel.ac.in/courses/Webcourse-contents/IIT%20Kharagpur/Power%20Electronics/New_index1.html
188 https://en.wikipedia.org/wiki/Electric_power_conversion
189 https://en.wikipedia.org/wiki/History_of_the_transistor

SS PE devices in converters often perform high speed switching,

### Diodes

Etymology: di=two + electrode.[190] There are many types of *diodes* P-N Junction diodes consist of 2 semiconductor layers, p and n. The diode's behavior in a circuit "is given by its *current–voltage characteristic*, or *I–V graph*… [image to follow] The shape of the curve is determined by the transport of charge carriers through the so-called depletion layer or depletion region that exists at the p–n junction between differing semiconductors…"[191]




The diode symbol (shown above on the left) includes an arrowhead on the positive anode side directed toward the negative cathode, and indicating the direction that conventional (positive charges) current passes freely. The *I–V graph* shows that when anode to cathode voltage across the diode $V_{AK}$ is biased negative, essentially no current (or just a trickle, $I_s$) flows until an excessive *breakdown voltage* $V_{BR}$ is reached, at which point a high avalanche current destroys the diode. When the diode is forward biased with positive $V_{AK}$ voltage, the diode at about 0.7 V becomes like an ON (closed) switch offering little resistance, and current flow is

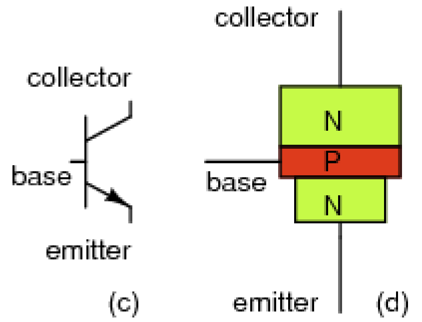
[190] http://www.electronicshub.org/types-of-diodes/ and https://en.wikipedia.org/wiki/Diode
[191] text and main graphic: https://en.wikipedia.org/wiki/Diode
graphic insert: http://www.allaboutcircuits.com/textbook/semiconductors/chpt-3/introduction-to-diodes-and-rectifiers/

determined mostly by the load resistance. Modern power diodes are constructed to handle currents into the 10 kA range, and when fabricated and/or used in series, can process voltages as high as 1 MV.[192]

Examples of diode use in Lab 1 are presented further below.

### Bipolar Junction Transistors BJT

These consist of 3 layers, N-P-N (common in power electronics, especially for high speed ON/OFF switching) or P-N-P layers. The middle Base layer is thin, thus the transistor differs from back-to-back diodes.[193] Bipolar transistors use "both electron and hole charge carriers. In contrast, unipolar transistors, such as field-effect transistors [FETs], only use one kind of charge carrier. For their operation, BJTs use two junctions between two semiconductor types, n-type and p-type."[194] BJT transistors offer high switching speeds and high reliability. The main drawbacks are (1) high internal heating in the saturation region when $I_C$ is very high, and (2) the transistor will remain closed (ON) only when maximum base current is present, causing high losses in the base circuit.[195]

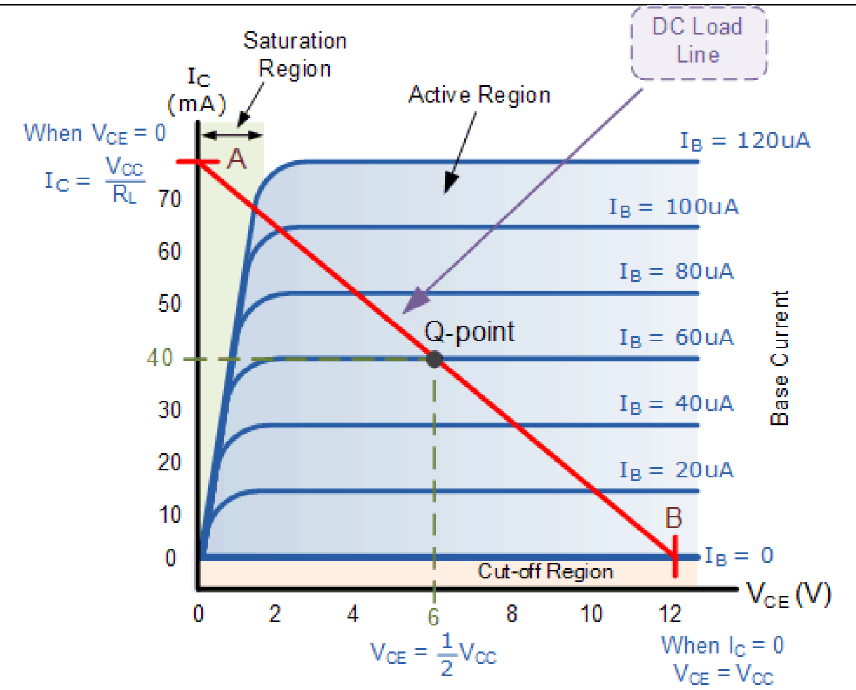



Schematic Diagram and structure of NPN BJT.[196] Note that the base layer is shown as relatively thin.

The voltage between the collector and emitter is $V_{CE}$, between base and collector is $V_{CB}$, and between base and emitter is $V_{BE}$.

A twin letter subscript (e.g., $V_{CC}$ or $V_{BB}$) indicates a power supply voltage source. Vcc is the collector supply; $V_{BB}$ is the base supply; $V_{EE}$ is the emitter supply. $V_{CC}$ is the voltage actually at the supply source which, after passing through the load resistor, reaches the collector.[197]

Conventional positive current $I_C$ enters at the collector C, current $I_B$ enters at the base B, and current $I_E$ exits at the emitter E.




The characteristic curves depicting $I_C$ vs. $V_{CE}$ show 3 regions.[198]

In the *cutoff region*, the base current is 0 and $I_C$ is nearly 0, so that the transistor is effectively switched OFF (switch open).

In the *saturation region*, the transistor is effectively switched ON (switch closed), $V_{CE}$ is nearly 0, and $I_C$ is determined primarily by the supply voltage $V_{CC}$, which typically passes through a load resistor $R_L$ to reach the collector.

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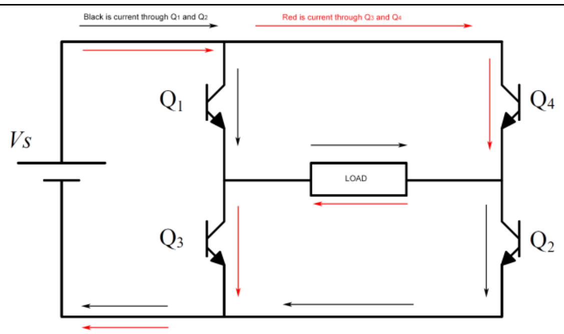
192 http://www.chtechnology.com/rectifiers.html and *EEAI3* p. 319
193 *EEAI3* p. 330
194 https://en.wikipedia.org/wiki/Bipolar_junction_transistor text and first image, by author:Inductiveload
195 *EEAI3* p. 322
196 http://www.allaboutcircuits.com/textbook/semiconductors/chpt-4/bipolar-junction-transistors-bjt/
197 http://www.electro-tech-online.com/threads/vcc-and-vbb.11361/
198 http://www.electronics-tutorials.ws/transistor/tran_2.html

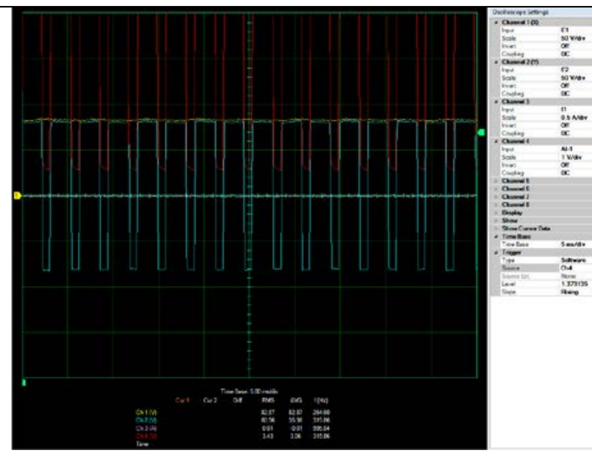
The following describes some interesting details about the region of operation useful in audio and video amplifiers but not usually in power electronics. In the *linear region* (aka *active region* or *mode*), the transistor acts as an amplifier , and $I_C = \beta I_B$. Power transistors cannot operate in this region long due to excessive heating. The DC current gain or amplification $\beta = I_C/I_B$ (also shown as $h_{FE}$ or $\beta_{DC}$) is almost constant for increasing $I_B$, and the amplification is nearly linear. β can be in the hundreds. The term $h_{FE}$ derives from the h-parameter model, 'F' is from forward current amplification, and 'E' refers to the transistor operating in a common emitter (CE) configuration. [199] "In amplifier design applications, the *Q-point* [quiescent point] corresponds to DC values for $I_C$ and that are about half their maximum possible values,,,. This is called *midpoint biasing* and it represents the most efficient use of the amplifiers range for operation with AC signals. The range that the input signal can assume is the maximum and no clipping of the signal can occur (assuming that the signal range is less than the maximum available)."[200] The base voltage is biased to operate halfway between its cut-off and saturation regions, thereby allowing the transistor amplifier to accurately reproduce the positive and negative halves of the highest amplitude AC input signal superimposed upon this positive DC biasing voltage.[201]

## H-Bridge circuit for DC to AC conversion

The following material from course Lab 1 and the textbook[202] demonstrates use of the NPN BJT to produce *DC to AC conversion* using a single phase H-Bridge circuit. "An H bridge is an electronic circuit that enables a voltage to be applied across a load in either direction. These circuits are often used in robotics and other applications to allow DC motors to run forwards and backwards."[203] According to lab TA John J. Sealy, this lab actually uses Insulated-gate bipolar transistors (IGBTs, see below).




**H-Bridge circuit for DC to Single Phase AC conversion:** The triggering components, which determine the switching period of the transistors, are not shown but implied. Black arrows indicate current flow passing through $Q_1$ and $Q_2$; Red arrows indicate current flow passing through $Q_3$ and $Q_4$. The red current direction through the load is the reverse of the black current flow through the load, thus providing a squarish type of AC which is not sinusoidal. The input $V_S$, triggering, and load voltages are graphed to the right.



Yellow (Ch1) is input DC voltage (a bit noisy) measuring 83V rms. Red square wave (Ch4) = triggering base voltage, peaks are off scale in this image but more than 4V peak, 3.4V rms, duty cycle appears asymmetrical [but I lack specifics], frequency is 315 Hz. Blue (Ch2) is output voltage of 80.6 V rms 316Hz AC.

An analogous DC to AC conversion can be done with conversion of a single DC source to 3-phase AC, using a six-pulse converter having 6 transistors.[204]

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[199] https://en.wikipedia.org/wiki/Bipolar_junction_transistor#h-parameter_model
[200] http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-071j-introduction-to-electronics-signals-and-measurement-spring-2006/lecture-notes/20_bjt_2.pdf
[201] http://www.electronics-tutorials.ws/transistor/tran_2.html paraphrased and interpreted
[202] *EEAI3* p. 351, schematic diagram from Lab 1 amended by MCM
[203] https://en.wikipedia.org/wiki/H_bridge
[204] *EEAI3* p. 352

## Pulse Width Modulation (PWM) for DC to AC conversion

This is another technique for converting DC to AC, in order to control the magnitude of the output voltage, the frequency of the output voltage, and the phase sequence of the output voltage.[205] The textbook *EEAI3* did not provide a circuit example (other than the 3-phase DC/AC inverter on p. 353), and I have not studied this topic in adequate detail.

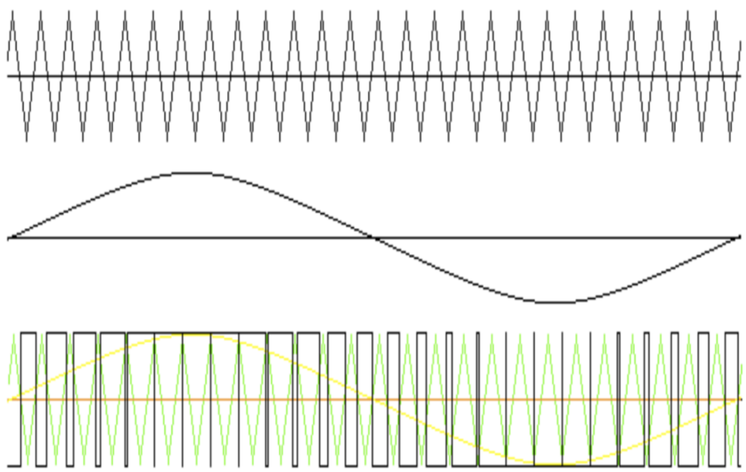
PWM should not be confused with PCM or Pulse-code modulation,[206] which is used to sample and encode (digitize) an analog audio signal as digital audio for use with CDs, DVDs, digital telephony, and other audio streams.

A carrier signal $v_{carrier}$ of fixed rms voltage and fixed high frequency is used, along with 3 control signals of adjustable frequency and voltage. (For single phase, only one control signal is needed.) The PWM circuit determines the switching status of the 6 transistors in the 3-phase DC/AC inverter (4 transistors in single phase). The control frequencies are shifted from each other by the requisite 120° for balanced 3-phase. In one PWM 3-phase technique, the switching of transistors $Q_1$ and $Q_4$, which join emitter to collector at point a, is determined as follows (where $\Delta v_a = v_{a\text{-control}} - v_{carrier}$):

If $\Delta v_a > 0$, $Q_1$ is closed and $Q_4$ is open
If $\Delta v_a < 0$, $Q_1$ is open and $Q_4$ is closed

The varying switching determines the duration of pulses (for example, pulses that alternate between voltage $v_{a0}$ or 0, where $v_{a0}$ is measured between point a and the negative side of the DC source). The varying pulse width is the basis for the name "width modulation". The line-to-line voltage between a and b is given by $v_{ab} = v_{a0} - v_{b0}$.

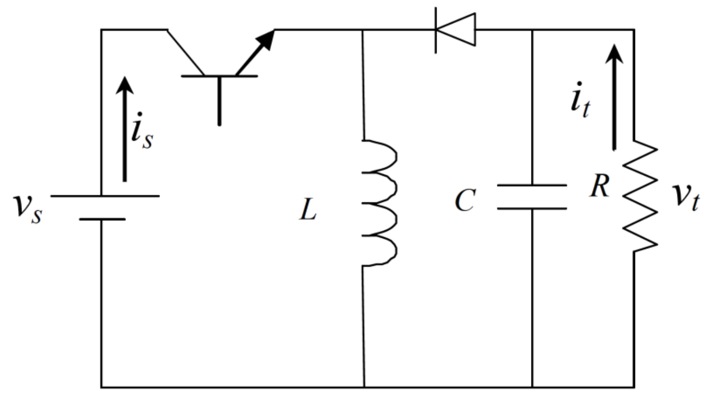
A graphic, adapted from an audio example, depicts the high frequency carrier (at top, a sawtooth pattern in this case), the width-modulated pulses (at the bottom, for example, $v_{ab} = v_{a0} - v_{b0}$), and the smoothed output AC waveform for one phase (in the middle, oscillating at the frequency of the control frequency):[207]



## Buck Boost DC to DC converter

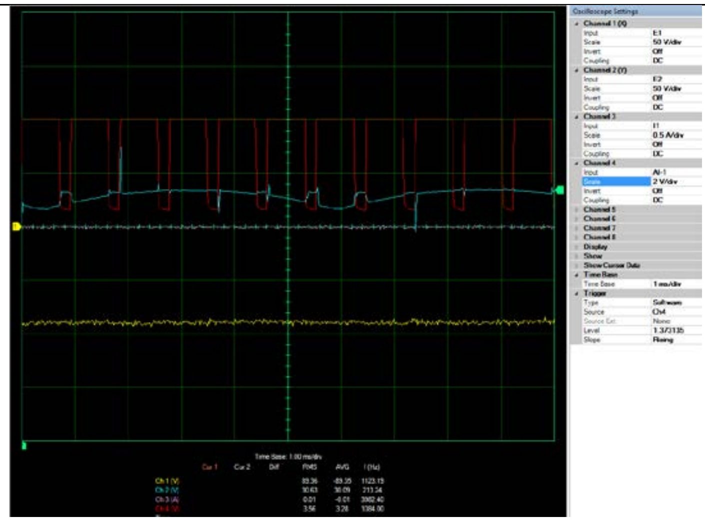
Another example of power conversion that was examined in the course Lab 1 is the buck-boost converter, which is used to perform DC to DC conversion (step-down or step-up voltages, accompanied by reversed polarity).[208] It makes use of both an NPN BJT as well as a conventional diode. (The text also describes the *buck converter* or *chopper*, for step-down, and the *boost converter*, for step-up.)

[205] *EEAI3* p. 356
[206] https://en.wikipedia.org/wiki/Pulse-code_modulation
[207] http://www.allaboutcircuits.com/textbook/semiconductors/chpt-11/pulse-width-modulation/ , modified slightly by MCM
[208] *EEAI3* p. 349




**Buck Boost DC to DC converter**

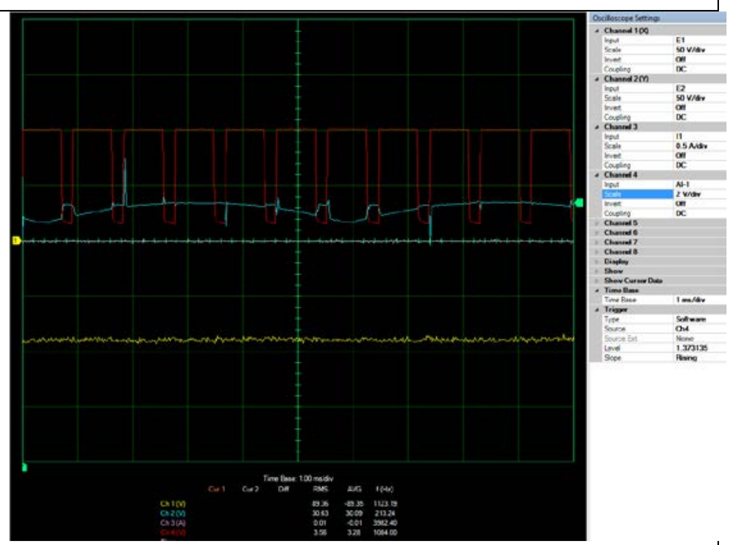
The transistor acts as a switch. When ON, energy flows into and is stored in the inductor, and blocked from the load resistor by the diode. When OFF, energy flows out of the inductor into the load (in the reverse polarity compared to the DC source, so current is allowed by the diode). The capacitor smoothes the fluctuations. The average value of the load voltage is controlled by adjusting the *duty ratio* (*duty cycle*), defined as ($t_{on}$ / $t_{on}$ + $t_{off}$), thus the fraction of commutation period τ for which the transistor is ON (closed).[209]



Yellow (Ch1) is input DC voltage (a bit noisy) measuring -89V average. Red square wave (Ch4) = triggering base voltage, peaks are at 4V peak, 3.6V rms, duty cycle is asymmetrical [but I lack specifics], frequency is 1084 Hz. Blue (Ch2) is output voltage of +30V average DC value (though varying irregularly at 213Hz). This is an example of both reversing polarity and reducing the absolute value of the magnitude of the varying DC voltage.

In the Buck Boost DC to DC converter example to the right:

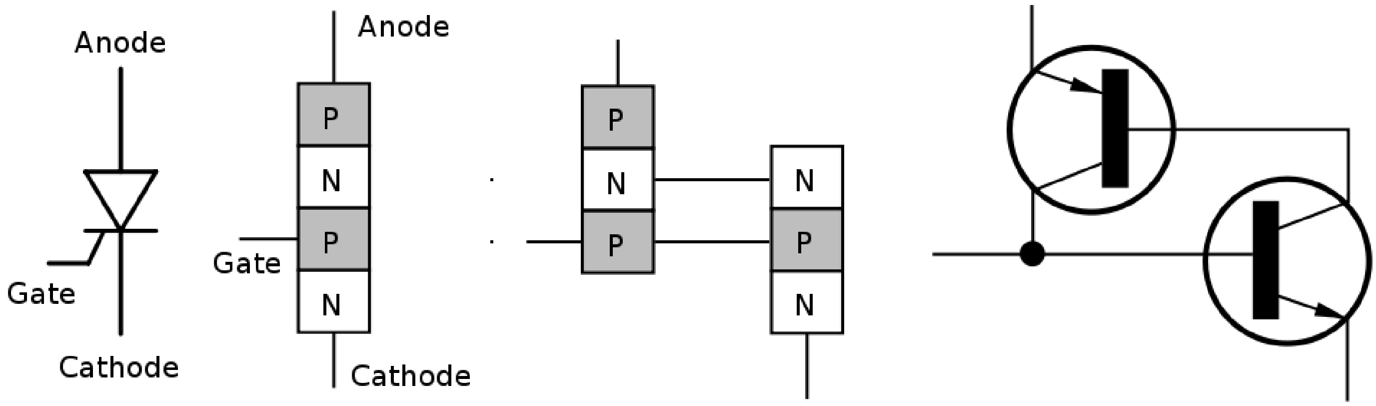
Yellow (Ch1) is input DC voltage (a bit noisy) measuring -19.4V average. Red square wave (Ch4) = triggering base voltage, peaks are at 4V peak, 2.5V rms, duty cycle is asymmetrical and differs from above [but again I lack specifics], frequency is same as above at 1084 Hz. Blue (Ch2) is output voltage of +33.7V average DC value (though varying irregularly). This is an example of both reversing polarity and increasing the absolute value of the magnitude of the varying DC voltage.



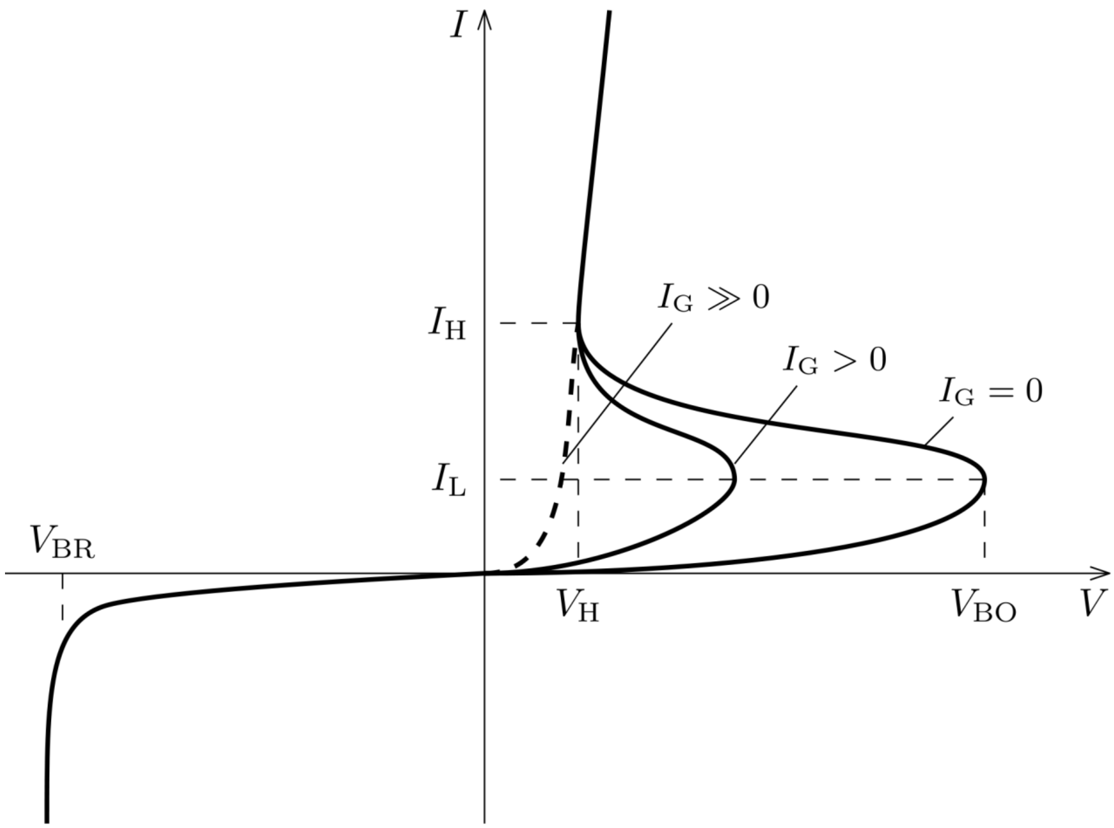
## Silicon Controlled Rectifiers SCR

This is a 4 semiconductor layer (PNPN) device consisting of P and N layers at the Anode A, then a Gate layer G, and finally an N layer at the Cathode (K). According to the textbook *EEAI3*, the SCR is a member of the larger family called *thyristors* (etymology: thura=gate + transistor).[210]

[209] https://en.wikipedia.org/wiki/Buck%E2%80%93boost_converter
[210] https://en.wikipedia.org/wiki/Thyristor




In the diagram above,[211] the diode symbol with Anode and Cathode as usual is supplemented by a gate electrode G. The 4 semiconductor layer PNPN structure is shown, with a physical diagram and an equivalent schematic depicting two tightly coupled *bipolar junction transistors* (BJTs). The upper BJT is has PNP layers, the lower has NPN.[212]




The *I–V graph* above depicts responses for thyristors including SCRs as follows (where V is the Anode to Cathode voltage and I is the Anode to Cathode current).[213]

> "In a conventional thyristor, once it has been switched ON [closed] by the gate terminal, the device remains latched in the ON-state [closed] (i.e. does not need a continuous supply of gate current to remain in the ON state), providing the anode current has exceeded the latching

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[211] https://en.wikipedia.org/wiki/Silicon_controlled_rectifier , adapted by MCM
[212] https://en.wikipedia.org/wiki/Bipolar_junction_transistor
[213] https://en.wikipedia.org/wiki/Thyristor text and image

current ($I_L$). As long as the anode remains positively biased, it cannot be switched OFF [open] until the anode current falls below the holding current ($I_H$).

A thyristor can be switched OFF [open] if the external circuit causes the anode to become negatively biased (a method known as natural, or line, commutation)…

After the current in a thyristor has extinguished, a finite time delay must elapse before the anode can again be positively biased and [yet] retain the thyristor in the OFF-[open] state. This minimum delay is called the circuit commutated turn off time ($t_Q$). Attempting to positively bias the anode within this time causes the thyristor to be self-triggered by the remaining charge carriers (holes and electrons) that have not yet recombined [, thus returning the thyristor to the ON closed state].

For applications with frequencies higher than the domestic AC mains supply (e.g. 50 Hz or 60 Hz), thyristors with lower [shorter] values of $t_Q$ are required… Today, fast thyristors are more usually made by electron or proton irradiation of the silicon, or by ion implantation…"

The negative voltage $V_{BR}$ is the reverse breakdown voltage—when attained, the device is destroyed, as with a diode. At the positive $V_{BO}$, the *breakover voltage*, the diode is switched ON (closed), even if there is no gate current. For progressively higher values of gate current, the SCR is latched ON at progressively lower but positive voltages $V_{AK}$. Once $I_H$ is attained, the device remains latched ON (closed), allowing a high current to flow determined by the load even for fairly low voltages $V_{AK}$.

## Single phase full wave rectification (AC to DC converter)

In the instructional lab, our class demonstrated the successful function of a full wave rectifier circuit employing 4 SCRs. The demos are implemented using LabVolt (now Festo Didactic) modules interfaced with a desktop computer. This equipment was demonstrated to me by TA John J. Sealy.

A single SCR can be used as a half-wave rectifier, but this is wasteful of power.

Circuit diagrams are shown immediately below for a single phase full wave rectifier and for a 3-phase full wave rectifier. These are examples of AC to DC converters, one of the major categories of power electronics. It should be emphasized that the *direct current* or *DC* may be highly variable and not constant, as long as it varies within one polarity, positive or negative. By varying the triggering (gating) angle α, the rms output voltage across the load can be varied from zero to $V_{max}/\sqrt{2}$, as shown in the formula:[214]

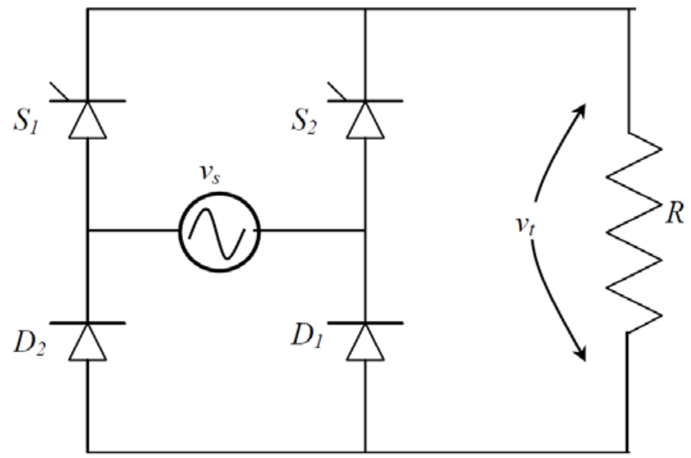
$$V_{\text{rms}-\text{full width}} = \frac{V_{\max}}{\sqrt{2}}\sqrt{\left(1-\frac{\alpha}{\pi}+\frac{\sin 2\alpha}{2\pi}\right)}$$

Usage of these circuits include *constant-current circuits*, often used to

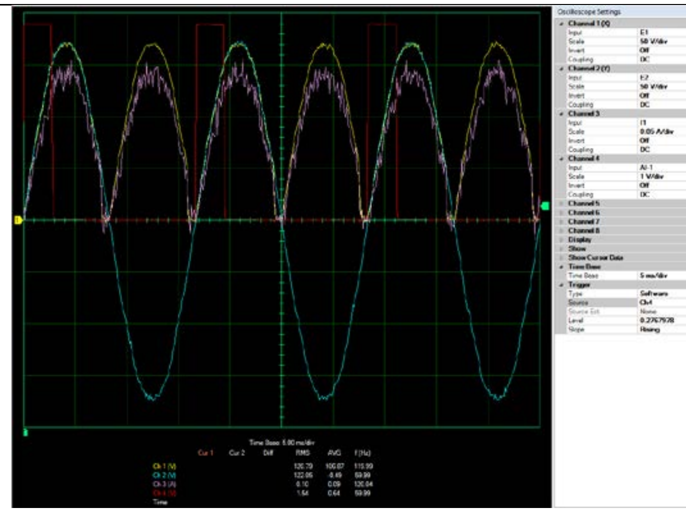
(1) charge batteries while protecting against excess charging current. (According to the text, these utilize a varying triggering angle α, which depends on the battery voltage. The current flows to charge the battery when $\alpha_{min} < \alpha < 180° - \alpha_{min}$. When battery voltage is low, average charging current is reduced by an increase in $\alpha_{min}$., and increase as the battery voltage rises.)[215] [Thus the charging current is not actually constant!]
(2) drive motors at constant torque.

While these demo circuits are instructive, real-world rectification and battery charger circuits would typically involve more complex control circuitry to achieve more uniform output voltage, safe charging current, etc.

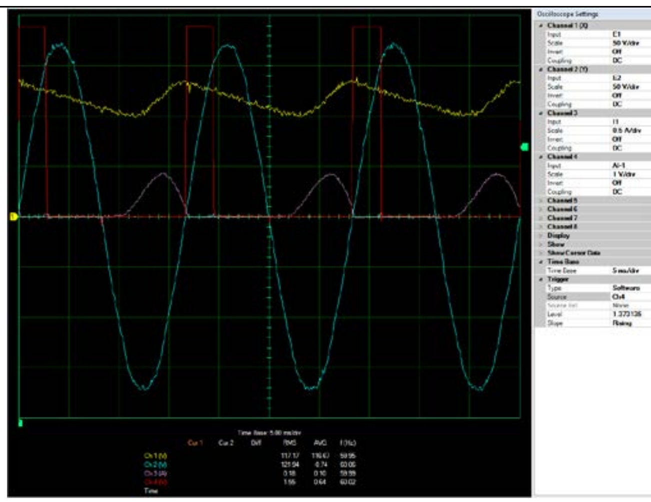
[214] *EEAI3* p. 335
[215] *EEAI3* p. 336



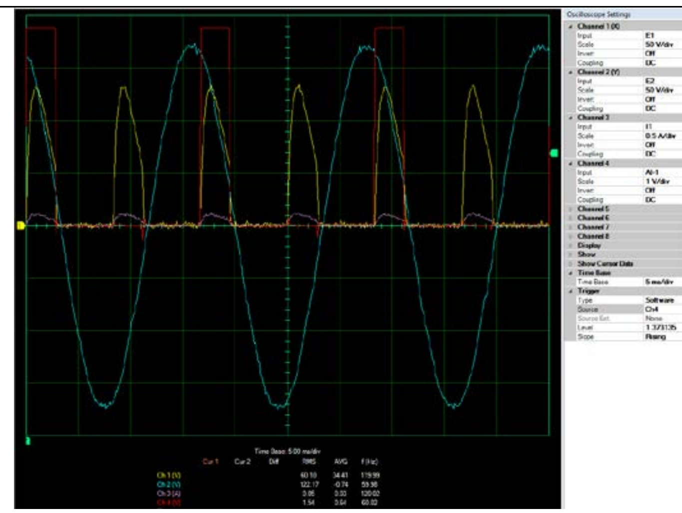

**Single phase full wave rectification**: Diagram, taken from the course's Lab 1 and the textbook,[216] depicts single phase full wave rectification using 2 SCRs and 2 diodes. The gating circuitry is implied but not shown.



Full wave rectification with SCR bridge shown to left and no LC smoothing.
Red square wave (Ch4) = triggering voltage, about 3.8V peak. Gating angle α = 0°
Blue (Ch2) is input voltage of 122V rms 60Hz AC.
Yellow (Ch1) is output rectified DC voltage, which is quite variable, is 121V rms at 120Hz.
Magenta is output current, 0.1 A rms at 120 Hz.

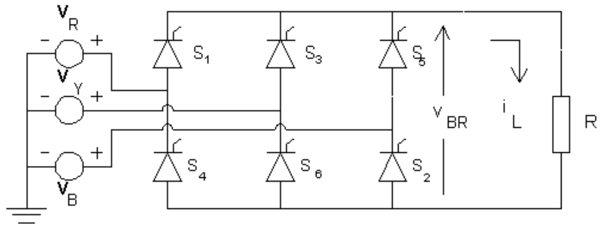


Full wave 1-phase rectification, but with output voltage smoothing resulting from adding a capacitor in parallel and an inductor in series.
Red square wave (Ch4) = triggering voltage, gating angle still α = 0°.
Blue (Ch2) is input voltage.
Yellow (Ch1) is output rectified DC voltage, now less variable with 117 V rms, 60Hz due to smoothing.
Magenta is output current, 0.18 A rms at 60 Hz, delayed apparently from LC effects.

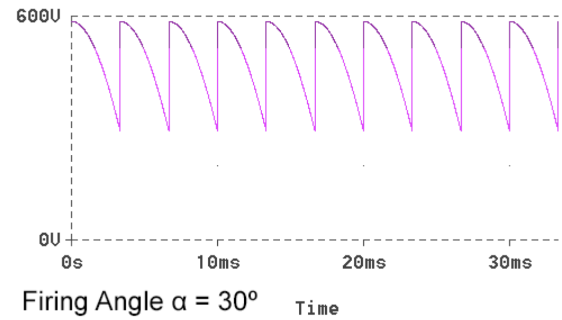


Full wave rectification , and with little or no output voltage smoothing.
Red square wave (Ch4) = triggering voltage, gating angle at about 120° (past the peak input voltage).
Blue (Ch2) is input voltage. Yellow (Ch1) is output rectified DC voltage, quite variable at 60V rms, 120Hz.
Magenta is output current, 0.05 A rms at 120 Hz, delayed due to large gating angle.

[216] *EEAI3* p. 334 (in chapter 10)




3-phase source (phases are represented by Red, Yellow, and Blue), full wave rectification via 6 SCRs, output presented as single phase to single load. The triggering circuitry for the SCRs is not shown.




Graph shows sample output to the load with firing angle α = 30 degrees and no LC smoothing of the waveform. "At any time two SCRs need to conduct, one from the top half and another [from the] bottom half… Two SCRs are triggered at the same time. For example, when SCR $S_2$ is to be triggered, SCR $S_1$ is also triggered. In the same way, when SCR $S_3$ is to be triggered, SCR $S_2$ is also triggered…"[217] There are 6 switching events per cycle,[218] with each event having a conduction period of 60º.

Other devices mentioned as relevant to Power Electronics include:

- Metal Oxide Semiconductor Field Effect Transistor (MOSFET): an efficient voltage-controlled device in which the ON (closed) input resistance ($V_{DS}/I_D$) is a few milliohms) [D=Drain; S = Source; G = Gate]
- Other Thyristors
  - Silicon Diode for Alternating Current (SIDAC). This voltage-triggered device switches ON (closed) at a fixed breakover voltage.
- Hybrid Power Devices:
  - Darlington Transistor: This combines two bipolar transistors to reduce base current in the saturation region, when compared to a single BJT.[219]
  - Insulated Gate Bipolar Transistor (IGBT): combine and provide the benefits of a MOSFET and a BJT.[220] "The IGBT combines the simple gate-drive characteristics of MOSFETs with the high-current and low-saturation-voltage capability of bipolar transistors."[221]

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[217] http://www.technik-emden.de/~elmalab/projekte/ws9899/pe_html/ch05s2/ch05s2p1.htm text & both images
[218] *EEAI3* p. 342
[219] *EEAI3* p. 327
[220] *EEAI3* p. 328
[221] https://en.wikipedia.org/wiki/Insulated-gate_bipolar_transistor

# Transformers

This discussion is partly drawn from the textbook Chapter 11.[222]

A transformer is "An electrical device without moving parts, which transfers energy of an alternating current in the primary winding to that in one or more secondary windings, through electromagnetic induction. Except in the case of the *autotransformer* there is no electrical connection between the two windings and, except for the *isolating transformer*, the voltage is changed."[223] It is also defined as, "a device employing the principle of mutual induction to convert variations of current in a primary circuit into variations of voltage and current in a secondary circuit".[224] A transformer may have more than two conductors or windings.

## Invention

Early ZBD toroidal iron core transformer from 1885[225]

Drawing on Michael Faraday's work with inductance in the 1820s and 1830s, including a device he created in 1831 which had all basic elements of a transformer,[226] "scientists discovered that, when two inductors were placed side by side without touching, the magnetic field from the first coil affects the secondary coil—this discovery led to the invention of the first transformers."[227] The transformer was invented by Antonio Pacinotti in 1860 according to the textbook,[228] though I found little about this in English and a photo only of his DC electrical generator or dynamo of 1860.

Others are also credited with being involved in its invention and development. The story is complex, and I can offer only a few key early events.

"… Ottó Bláthy, Miksa Déri, Károly Zipernowsky of the Austro-Hungarian Empire first designed [at the Ganz Company] and used the [ZBD toroidal shaped] transformer in both experimental, and commercial systems. Later on Lucien Gaulard, Sebstian Ferranti, and William Stanley perfected the design…

The property of induction was discovered in the 1830's but it wasn't until 1886 [or 1885] that William Stanley, working for Westinghouse built the first reliable commercial transformer. His work was built upon some rudimentary designs by the Ganz Company in Hungary (ZBD Transformer 1878), and Lucien Gaulard and John Dixon Gibbs in England. Nikola Tesla did not invent the transformer… The Europeans mentioned above did the first work in the field. George Westinghouse, Albert Schmid, Oliver Shallenberger and Stanley made the transformer cheap to produce, and easy to adjust for final use."[229]

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[222] *EEAI3* p. 363-394
[223] http://thesciencedictionary.org/transformer/
[224] http://www.merriam-webster.com/dictionary/transformer
[225] http://www.edisontechcenter.org/Transformers.html , image retouched by MCM
[226] http://ieeexplore.ieee.org/iel1/39/11919/00546444.pdf 1996 article. requires pmt or university proxy
[227] http://www.cettechnology.com/history-of-transformers-and-inductors/
[228] *EEAI3* p. 363
[229] http://www.edisontechcenter.org/Transformers.html , including ZBD transformer photo slightly enhanced by MCM

Another source states, "Those credited with the invention of the transformer include:[230]

- Lucien Gaulard and John Gibbs, who first exhibited a device in London in 1881 and then sold the idea to the American company Westinghouse. They also exhibited the invention in Turin in 1884, where it was adopted for an electric lighting system.
- William Stanley of Westinghouse, who built the first model based on Gaulard and Gibbs' idea in 1885.
- In 1884-85, three Ganz Factory engineers, Ottó Bláthy, Miksa Déri and Károly Zipernowsky, developed a new power distribution system, based on the application of transformers.
- ...Nikola Tesla's "... true achievement was to develop the device for practical AC power distribution."

Regarding the ZBD inventors: "On Bláthy's suggestion, transformers were constructed with a closed iron core; their joint work resulted in one of the most important inventions in electro-technology at that time. This system is still the basis of long-distance power transfer and high-voltage electric energy distribution. In 1885 they took out a patent relating to *alternating current closed iron core transformers*."[231] The image above depicts an early toroidal iron core ZBD transformer from 1885.[232]

## Types and Uses of Transformers

Definitions are given here with some overlap due to varying sources:

Power systems involve transformers that step up and step down voltage, and include the following:[233]

**Transmission transformer**: "Power transformers are used in transmission network of higher voltages for step-up and step down application (400 kV, 200 kV, 110 kV, 66 kV, 33kV) and are generally rated above 200MVA..." The high voltages reduce current and therefore $I^2R$ energy losses as heat.

**Distribution transformer**:[234] Installed in distribution substations located near load centers, these reduce transmission line high voltage to 5 to 220 KV for local distribution ending at service transformers.

**Service transformer**: These are located close to customer's loads, and reduce distribution voltage to split phase 120/240V (in the US) for final delivery to the customer loads.

Power systems also use transformers for less raw power applications:

**Circuit transformer**: Small transformers used in power supplies and electronic circuits. These can also be used for *impedance matching*, *filters*, and *electrical isolation* of different parts of a circuit.

Some of the specific transformer types, including non-power applications, include:[235]

**Autotransformer**: Transformer in which part of the winding is common to both primary and secondary circuits. "An autotransformer has only a single winding, which is tapped at some point along the winding. AC or pulsed DC power is applied across a portion of the winding, and a higher (or lower) voltage is produced across another portion of the same winding. Autotransformers are commonly used as spark coils in automotive engines, and as high-voltage flyback transformers in television sets and computer monitors... Variac was a trademark in the mid-20th century for a *variable autotransformer* intended to conveniently vary the output voltage for a steady AC input voltage. A sliding contact determined what fraction of the winding was connected across the output; a common configuration provided for 120 V as input and percentages of

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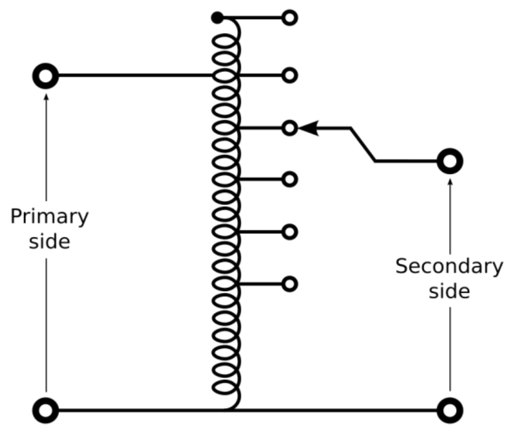
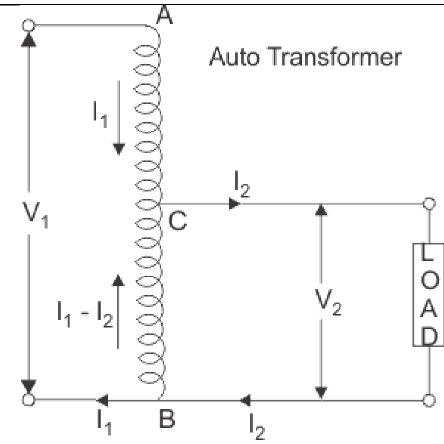
[230] http://www.fact-index.com/t/tr/transformer.html
[231] http://www.omikk.bme.hu/archivum/angol/htm/blathy_o.htm
[232] http://www.edisontechcenter.org/Transformers.html *
[233] *EEAI3* p. 363
[234] https://en.wikipedia.org/wiki/Distribution_transformer
[235] https://en.wikipedia.org/wiki/Transformer and https://en.wikipedia.org/wiki/Transformer_types

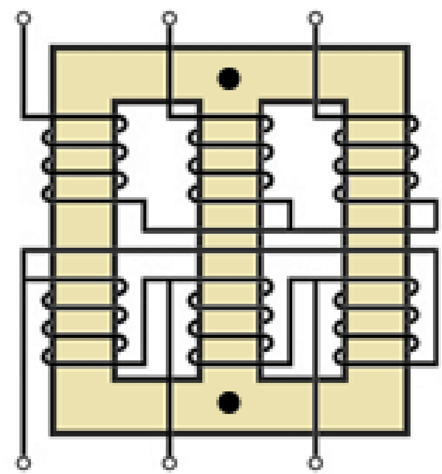
that voltage as high as about 110%. More compact semiconductor light dimmers have displaced them in many applications, such as theatrical lighting."[236]




Autotransformer with fixed step-down and step-up taps[237]




Autotransformer, depicting source and load voltages and currents[238]



**Three-Phase or Polyphase transformer**: Transformer with more than one phase (typically 3-phase). 3-phase may have Wye-Wye, Wye-Delta, Delta-Wye, or Delta-Delta configurations. "The reasons for choosing a Y or Δ configuration for transformer winding connections are the same as for any other three-phase application: Y connections provide the opportunity for multiple voltages, while Δ connections enjoy a higher level of reliability (if one winding fails open, the other two can still maintain full line voltages to the load)."[239]

The diagram to right depicts a 3-phase transformer with a star (Wye) connected set of windings and a delta connected set (i.e., Wye-Delta configuration). Here the core consists of three limbs that are closed at the top and bottom. Note that as a result of the properties of balanced 3 phase construction, including primary currents and resulting magnetic fluxes, the core fluxes induced by the 3 primary windings sum to 0 at the black dots I have added.[240] A *Phase angle regulating transformer* is a specialized transformer used to control the flow of real power on three-phase electricity transmission networks.

**Grounding transformer**: Transformer used for grounding three-phase circuits to create a neutral in a three wire system, using a wye-delta transformer, or more commonly, a zigzag grounding winding.

**Leakage transformer**: Transformer that has loosely coupled windings. "A leakage transformer, also called a *stray-field transformer*, has a significantly higher leakage inductance than other transformers, sometimes increased by a magnetic bypass or shunt in its core between primary and secondary, which is sometimes adjustable with a set screw. This provides a transformer with an inherent current limitation due to the loose coupling between its primary and the secondary windings. The output and input currents are low enough to prevent thermal overload under all load conditions—even if the secondary is shorted."[241] "This is particularly

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236 http://www.fact-index.com/t/tr/transformer.html and *EEAI3* p. 372
237 https://upload.wikimedia.org/wikipedia/commons/thumb/c/c5/Tapped_autotransformer.svg/960px-Tapped_autotransformer.svg.png
238 http://www.electrical4u.com/what-is-auto-transformer/ Discusses pros and cons of these.
239 http://www.allaboutcircuits.com/textbook/alternating-current/chpt-10/three-phase-transformer-circuits/
240 http://myelectrical.com/notes/entryid/199/power-transformers-an-introduction image modified by MCM
241 https://en.wikipedia.org/wiki/Transformer_types

required in the case of welding transformers, where high currents may occur if the welding electrode gets fused with the job. Thus these transformers have high leakage inductance."[242]

**Resonant transformer**: Transformer that uses resonance to generate a high secondary voltage. "A resonant transformer is a transformer in which one or both windings has a capacitor across it and functions as a tuned circuit. Used at radio frequencies, resonant transformers can function as high Q_factor bandpass filters. The transformer windings have either air or ferrite cores and the bandwidth can be adjusted by varying the coupling (mutual inductance). One common form is the IF (intermediate frequency) transformer, used in superheterodyne radio receivers. They are also used in radio transmitters… Resonant transformers are also used in electronic ballasts for fluorescent lamps, and high voltage power supplies…"[243]

**Audio transformer**: Transformer used in audio equipment. "Signal and audio transformers are used to couple stages of amplifiers and to match devices such as microphones and record players to the input of amplifiers. Audio transformers allowed telephone circuits to carry on a two-way conversation over a single pair of wires. A *balun (balun transformer*) converts a signal that is referenced to ground to a signal that has balanced voltages to ground, such as between external cables and internal circuits."

Chokes: These are not transformers but are of interest and are sometimes confused with transformers. Audio or radio frequency *chokes* are wound coils which have impedance which rises with frequency, thus they selectively impeded or block frequencies above a certain threshold value. A radio frequency choke provides *RF filtering*. Chokes may be similar in design to transformers. Chokes (*ferrite lumps or chokes*) are often placed along power cords to computer to filter out higher frequencies, reducing EMI and RFI propagation and broadcasting from the computer.[244]

Lamp Chokes: A *magnetic, coil, or choke ballast* is another wound coil similar to a choke but used for an entirely different function. Fluorescent lamps, mercury vapor lamps, sodium vapor lamps, iodine lamps, neon sign lamps … need a high voltage for starting and once started, the running [load] voltage assumes a value much lower than the mains voltage. The ballast provides the high voltage for starting."[245] "Fluorescent lamps are negative differential resistance devices, so as more current flows through them, the electrical resistance of the fluorescent lamp drops, allowing for even more current to flow. Connected directly to a constant-voltage power supply, a fluorescent lamp would rapidly self-destruct due to the uncontrolled current flow. To prevent this, fluorescent lamps must use an auxiliary device, a ballast, to regulate the current flow through the lamp."[246] However, the newer generation ballasts for fluorescent lighting are electronic, not magnetic, and operate at high frequencies.)

**Output transformer**: Transformer used to match the output of a tube amplifier, such as for an electric guitar, to its load, or to optimize the output regarding the quality of the sound. Leo Fender inhibited the low frequency response of his guitar amplifier, reducing the likelihood of speaker damage, by diminishing the actual size of the output transformer.[247]

**Instrument transformer**: *Potential transformer* or *current transformer* used to accurately and safely represent voltage, current or phase position of high voltage or high power circuits. "The most common usage of instrument transformers is to operate instruments or metering from high voltage or high current circuits, safely isolating secondary control circuitry from the high voltages or currents. The primary winding of the transformer is connected to the high voltage or high current circuit, and the meter or relay is connected to the secondary circuit."[248] The textbook includes a photo of a current transformer and diagram,[249] and one for 110 kV operation is depicted to the right.[250]

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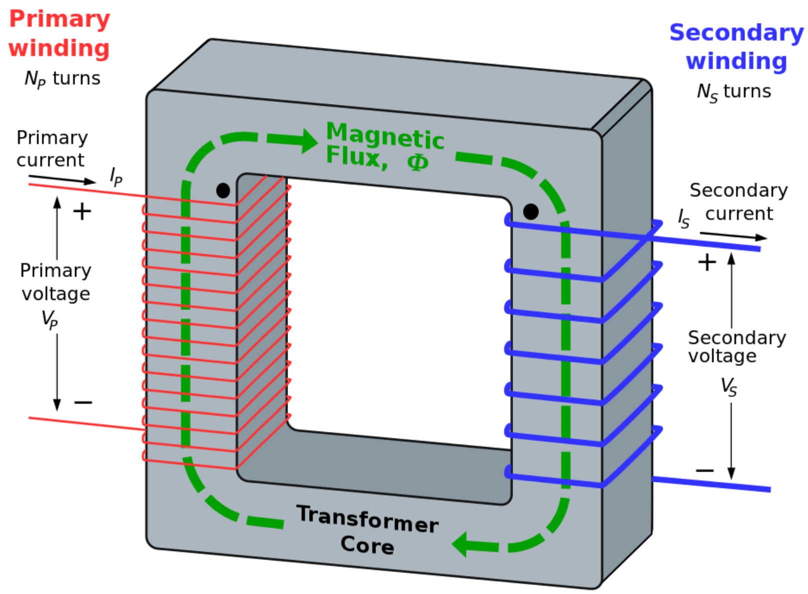
[242] http://www.electrotechnik.net/2011/02/leakage-transformers.html
[243] https://en.wikipedia.org/wiki/Transformer_types#Resonant_transformer
[244] https://audiosystemsgroup.com/SAC0305Ferrites.pdf
[245] https://answers.yahoo.com/question/index?qid=20071129091306AAHqlYI
[246] https://en.wikipedia.org/wiki/Fluorescent_lamp
[247] http://www.victoriaamp.com/understanding-your-output-transformer-part-one-by-mark-baier/
[248] https://en.wikipedia.org/wiki/Instrument_transformer
[249] *EEAI3* p. 30
[250] https://en.wikipedia.org/wiki/Current_transformer

**Pulse transformer**: Specialized small-signal transformer used to transmit digital signaling while providing electrical isolation, commonly used in Ethernet computer networks…[251]

## Transformer Construction and Theory




A representative diagram of a single phase transformer is shown to the right.[252] The windings are made of insulated wire and/or wire separated by insulating varnish, etc., so there is electrical isolation from the core

The primary winding carries AC current $I_p$ due to applied primary AC voltage $V_p$. The oscillating current in the primary winding induces an oscillating magnetic flux φ in the core (ideal instantaneous quantities, simplified):

$$V_p = +N_p \frac{d\varphi}{dt}$$

The instantaneous flux in the core is given by

$$\varphi = \frac{1}{N_1} \int V_p dt$$

φ is the magnetic field or flux density, which is proportional to the number of turns in the primary $N_p$. These values assume that all flux generated in the windings is captured in the core (infinite permeability), that there are no losses, and no net magnetomotive force.

For the depiction in the diagram above of current in the primary winding, where voltage is positive and rising and conventional current flows toward the core at the top of the primary winding, the magnetic flux points up in the direction of the green arrow on the primary side. This flux direction is given by applying Ampère's circuital law using the *right hand rule* for the case where a voltage induces a flux.[253] Correspondingly, the direction of current in the secondary as depicted reflects the application of Faraday's law of Induction wherein a flux induces a voltage:

$$V_S = -N_S \frac{d\varphi}{dt}$$

Here the flux on the secondary side points downward, and the right hand rule would give the current flowing toward the core at the top, but the minus sign indicates the current flows in the reverse direction, thus away from the core at the top, as depicted. (There is some confusion about formula signage. However, Lenz's law states that "If an induced current flows, its direction is always such that it will oppose the change which produced it."[254])

Diagrams of transformers sometimes use dots to show points where the primary and secondary potentials are in phase. (I have added these dots in the example above, based on a diagram in the textbook.[255] "Positively increasing instantaneous current entering the primary winding's dot end induces positive polarity voltage at the secondary winding's dot end."[256]

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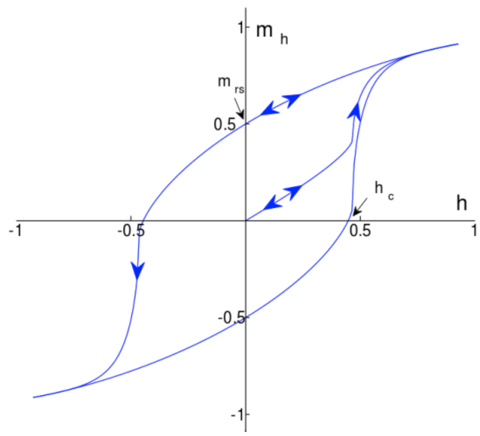
[251] http://www.encyclopedia-magnetica.com/doku.php/pulse_transformer
[252] https://en.wikipedia.org/wiki/Transformer
[253] https://en.wikipedia.org/wiki/Right-hand_rule#Direction_associated_with_a_rotation
[254] https://en.wikipedia.org/wiki/Lenz's_law
[255] *EEAI3* p. 369
[256] https://en.wikipedia.org/wiki/Transformer

**Core Construction**: The modern transformer core is made of high permeability silicon steel, which reduces the magnetizing current required and helps confine the flux to the core (minimizing *leakage*) in order to maximize coupling of the windings.[257] There are many important aspects of core design, and I mention only some of the considerations.

*Permeability* μ is a measure of the ability of a material to support the formation of a magnetic field within itself. It is the degree of magnetization (symbol usually **M**) induced in a material in response to an applied magnetic field (aka, auxiliary magnetic field, symbol usually **H**). Magnetic permeability is typically represented by the Greek letter μ. Thus, **B** = μ**H**. The reciprocal of magnetic permeability is *magnetic reluctivity* (symbol usually □ = 1/μ)… In SI units, permeability is measured in henries per meter ($H \cdot m^{-1}$) or $N \cdot A^{-2}$. The permeability constant ($\mu_0$), also known as the *permeability of free space*, is exactly $4\pi \times 10^{-7}$ $H \cdot m^{-1}$. the relative permeability is $\mu_r = \mu/\mu_0$. A closely related property of materials is *magnetic susceptibility*, which is a dimensionless proportionality factor that indicates the degree of magnetization of a material in response to an applied magnetic field, and is given by $X_m = \mu_r - 1$.[258]




In choosing the material for the core, the main problems are [magnetic] hysteresis loss and eddy current loss. For hysteresis (see graph[259], in which M is represented by $m_h$ and H is shown as h), the smaller the area enclosed in the hysteresis loop, the lower the *hysteresis loss*. A small quantity of silicon alloyed with low carbon content steel produces core material which has low hysteresis losses and high permeability. To further reduce the core losses, another technique known as *cold rolling* is employed. This technique reorients the grains in ferromagnetic steel in the direction of rolling. Steel with silicon alloying and cold rolling is commonly known as *Cold Rolled Grain Oriented Silicon Steel (CRGOS)*. This material is now universally used for manufacturing transformer core.[260] (*Cold-rolled non-grain-oriented steel,* often abbreviated CRNGO, is also used because of lower cost.) Core steels in general are much more expensive than mild steel.

CRGOS is also called *electrical steel*. "Electrical steel is an iron alloy which may have from zero to 6.5% silicon (Si:5Fe). Commercial alloys usually have silicon content up to 3.2% (higher concentrations usually provoke brittleness during cold rolling). Manganese and aluminum can be added up to 0.5%… Silicon significantly increases the electrical resistivity of the steel, which decreases the induced eddy currents and narrows the hysteresis loop of the material, thus lowering the core loss. However, the grain structure hardens and embrittles the metal, which adversely affects the workability of the material, especially when rolling it. When alloying, the concentration levels of carbon, sulfur, oxygen and nitrogen must be kept low, as these elements indicate the presence of carbides, sulfides, oxides and nitrides. These compounds, even in particles as small as one micrometer in diameter, increase hysteresis losses while also decreasing magnetic permeability. The presence of carbon has a more detrimental effect than sulfur or oxygen. Carbon also causes magnetic aging when it slowly leaves the solid solution and precipitates as carbides, thus resulting in an increase in power loss over time. For these reasons, the carbon level is kept to 0.005% or lower. The carbon level can be reduced by *annealing* the steel in a decarburizing atmosphere, such as hydrogen."[261]

Although CRGOS has low specific iron loss, it has some disadvantages: it is susceptible to increased loss due to flux flow in directions other than grain orientation, and it also susceptible to impaired performance due to impact of bending and cutting of the CRGOS sheet.

The core itself is like a secondary coil of one turn, and electrical *eddy currents* are induced in it which reduce efficiency and generate heat. In order to minimize these effects, the core is made of laminated plates separated by insulating varnish or low conducting oxidized layers to reduce electrical conduction. These

[257] https://en.wikipedia.org/wiki/Transformer
[258] https://en.wikipedia.org/wiki/Permeability_%28electromagnetism%29 edited and paraphrased
[259] https://en.wikipedia.org/wiki/Magnetic_hysteresis
[260] http://www.electrical4u.com/core-of-transformer-and-design-of-transformer-core/ paraphrased & edited
[261] https://en.wikipedia.org/wiki/Electrical_steel

laminations confine eddy currents to highly elliptical paths that enclose little flux and thus result in low eddy currents—the thinner the better, but costs rise with thinner plates. Cores made of powdered iron or non-conducting magnetic ferrites are sometimes used in higher frequency non-power applications.[262] Other expensive materials to reduce core eddy current losses, for instance in distribution transformers, include amorphous (non-crystalline) metal such as Metglas®.[263] It is possible to have a transformer with only an *air core*, sometimes used in RF applications.

Cores are often rectangular and either of core type or shell type in design (in the latter, the core partly surrounds the windings). A simple construction approach is to have E cross section laminations which are stacked and wound, and then capped with I-shaped laminations. (This is the so-called *E-I transformer core*.) Perhaps the ideal shape of the steel core is a toroid with a circular cross section (like a donut), but this makes the laminations vary in width and difficult to construct, and the toroid is more difficult to wind. *Oil cooling ducts* add to core complexity.

The limbs of the core laminations are typically layered in interleaved manner, which yields increased magnetic flux loss at the joints but lower manufacturing cost. A better arrangement with lower cross grain loss due to a smoother path for the flux is to have the core limb joints mitered at 45°, but costs are higher.[264] When transformer cores are assembled by joining up 2 or more sections after windings are applied (e.g., two C-shaped or E-shaped sections), any air gaps between the apposed sections greatly increase flux losses.

The oscillating magnetic flux induces a voltage (electromotive force EMF) in the secondary winding, given by

$$V_s = -N_2 \frac{d\varphi}{dt}$$

where the negative sign indicates that the flux induces $V_s$.

"Power-frequency transformers may have taps at intermediate points on the winding, usually on the higher voltage winding side, for voltage adjustment… Automatic on-load tap changers are used in electric power transmission or distribution, on equipment such as arc furnace transformers, or for automatic voltage regulators for sensitive loads. Audio-frequency transformers, used for the distribution of audio to public address loudspeakers, have taps to allow adjustment of impedance to each speaker…"[265]

A useful relation comparing voltages and number of windings is

$$V_T = \frac{V_P}{N_P} = \frac{V_S}{N_S} \text{ or } \frac{V_P}{V_S} = \frac{N_P}{N_S} \text{ or } V_S = V_P \frac{N_S}{N_P}$$

where $V_T$ is the voltage per turns in either winding. Therefore, if $N_S > N_P$, the transformer is a step-up voltage transformer whereas if $N_S < N_P$ , the transformer is step-down voltage transformer.

Assuming no losses, the output power is equal to the input power, the current ratios are given by

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_1}$$

The *magnetomotive forces* $\mathfrak{I}$ of the primary and secondary windings, expressed in ampere-turns, are equal and given by

$$\mathfrak{I} = I_1 N_1 = I_2 N_2$$

Thus, the current is smaller in the winding with a higher number of turns, and the wire in such a winding can be correspondingly smaller in caliber (having a higher wire gauge number).

Calculations of magnitudes are simplified by the concept of *Reflected Load Impedance*. Briefly, the load across the secondary $Z_{load} = E_2/I_2$. This load may be considered as "seen" from the primary by the following equation:

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[262] https://en.wikipedia.org/wiki/Transformer
[263] https://en.wikipedia.org/wiki/Amorphous_metal_transformer and http://metglas.com/products/magnetic_materials/2714a.asp
[264] http://www.electrical4u.com/core-of-transformer-and-design-of-transformer-core/
[265] https://en.wikipedia.org/wiki/Transformer

Reflected load impedance $Z'_{load} = Z_{load}\left(\frac{N_1}{N_2}\right)^2$

also called the load impedance referred to the primary winding.

## Transformer Ratings

A typical set of transformer ratings would be "10 kVA, 8kV/240V". Magnitudes thus are implied or may be derived as follows:

Rated apparent power S = 10 kVA

Voltage Ratio $V_P/V_S$ = 8000/240 = 33.3

Rated current of the primary $I_P = S/V_P$ = 10000/8000 = 1.25 A

Rated current of the secondary $I_S = S/V_S$ = 10000/240 = 41.7 A

At *full load*, when current and voltage are operating at (maximal allowable) rated values, load impedance is

$Z_{load} = V_2/I_2$ = 240/41.7 = 5.76 Ω

Magnitude of the reflected impedance of full load is given by

$$Z'_{load} = Z_{load}\left(\frac{N_1}{N_2}\right)^2 = 5.76(33.33)^2 = 6400\ \Omega = 6.4\ k\Omega$$

A representative rating for a 3-phase Delta-Wye transformer might be: "60 kVA, 8kV(Δ)/416V(Y)"

Rated apparent combined power of the 3 phases S = 60 kVA

Rated apparent power of each phase = 60/3 = 20 kVA

Primary windings are connected in Delta

Rated line-to-line voltage $V_{L-L}$ of primary circuit = 8 kV.

The voltage across any primary winding = 8 kV.

Secondary windings are connected in Wye

Rated line-to-line [$V_{L-L}$] voltage of secondary circuit = 416 V.

The voltage across any secondary winding [$V_{L-N}$] = 416/√3 = 240 V.

The line-to-line voltage ratio [$V_{primary\ L-L}/V_{secondary\ L-L}$] of the transformer is 8000/416 = 19.23

The voltage per turn is constant in any winding of the transformer, and given as

$$V_T = \frac{V_{phase\ of\ primary}}{N_1} \text{ etc.}$$

## Multiple Windings and Other Considerations

Transformers may have multiple windings. For transformers with multiple windings, the relationships again apply:

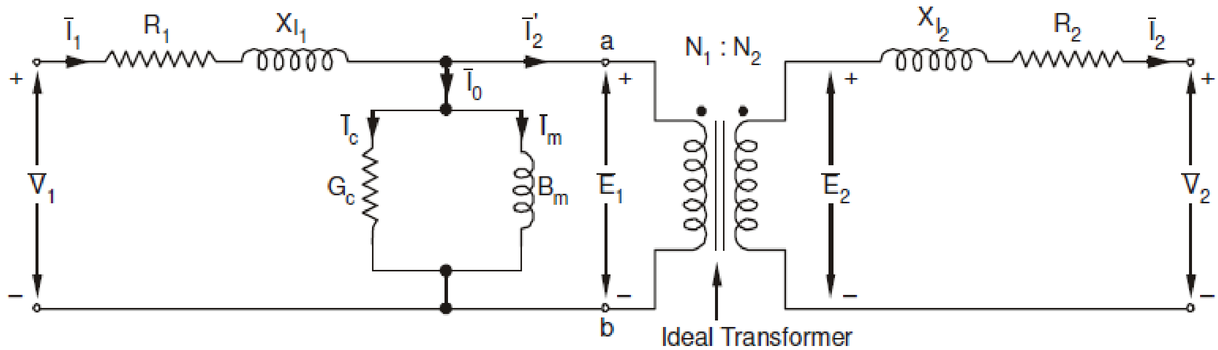
$$V_T = \frac{V_1}{N_1} = \frac{V_2}{N_2} = \frac{V_3}{N_3} \text{ etc.}$$

The computation of currents is more involved.[266]

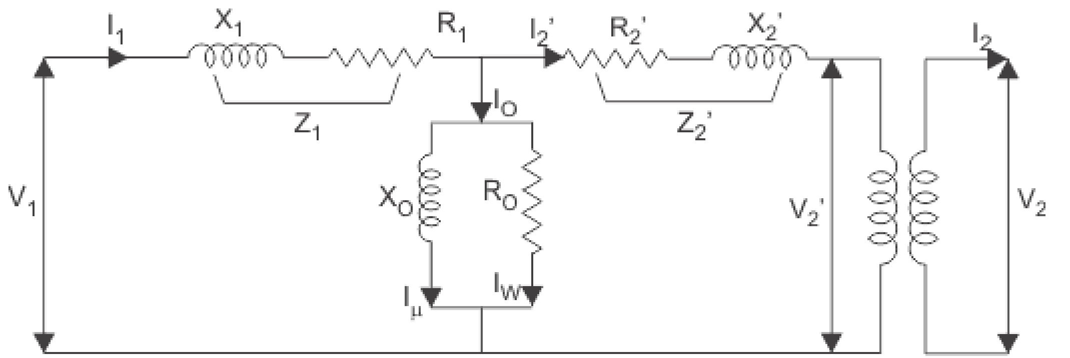
[266] *EEAI3* p. 370

For calculations with actual non-ideal transformers, it is necessary to consider non-zero parallel and series resistances and inductances in the primary and secondary circuits, B-H hysteresis effects, etc. (Details mostly omitted).

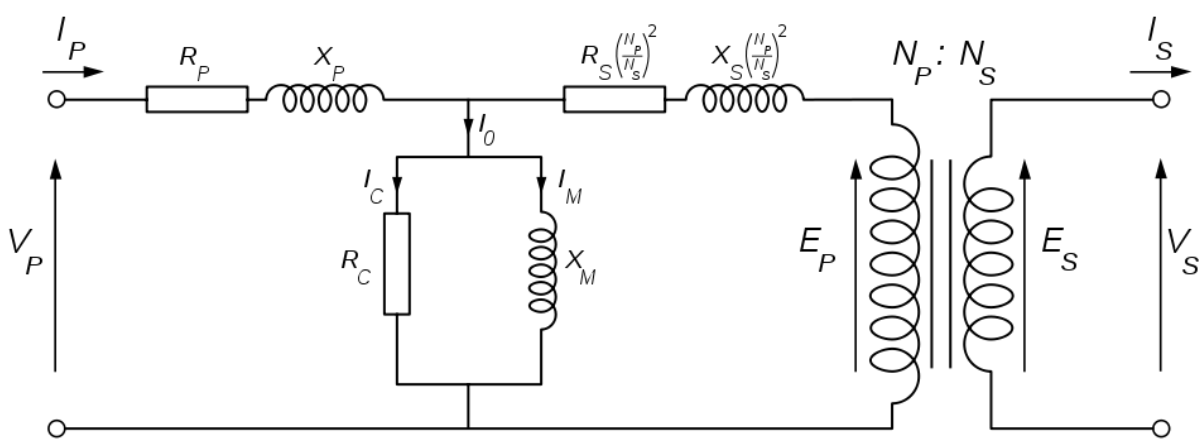
Equivalent circuits for representing non-ideal transformers include the following 3 examples:[267]




(1)




Equivalent Circuit of Transformer referred to Primary (2)




(3)

*Transformer Efficiency* η is given by one of several equivalent ratios:

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output power}}{\text{Output power+losses}} \text{ etc.}$$

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[267] Images are from these resp. sources:
(1) http://blog.oureducation.in/equivalent-circuits-of-transformer/
(2) http://www.electrical4u.com/equivalent-circuit-of-transformer-referred-to-primary-and-secondary/
(3) https://commons.wikimedia.org/wiki/File:Transformer_equivalent_circuit.svg

*Voltage Regulation* is another useful transformer index.

$$\text{Voltage Regulation} = \frac{|V_{\text{no load}}| - |V_{\text{full load}}|}{|V_{\text{full load}}|}$$

*Magnetostriction related transformer hum*: "Magnetic flux in a ferromagnetic material, such as the transformer core, causes it to physically expand and contract slightly with each cycle of the magnetic field, an effect known as *magnetostriction*, the frictional energy of which produces an audible noise known as mains hum or transformer hum. This transformer hum is especially objectionable in transformers supplied at power frequencies and in high-frequency flyback transformers associated with PAL system CRTs."[268]

[268] https://en.wikipedia.org/wiki/Transformer edited MCM

# Electric Machines (Motors and Generators)

This discussion is partly drawn from the textbook Chapter 12.[269]

## Motors

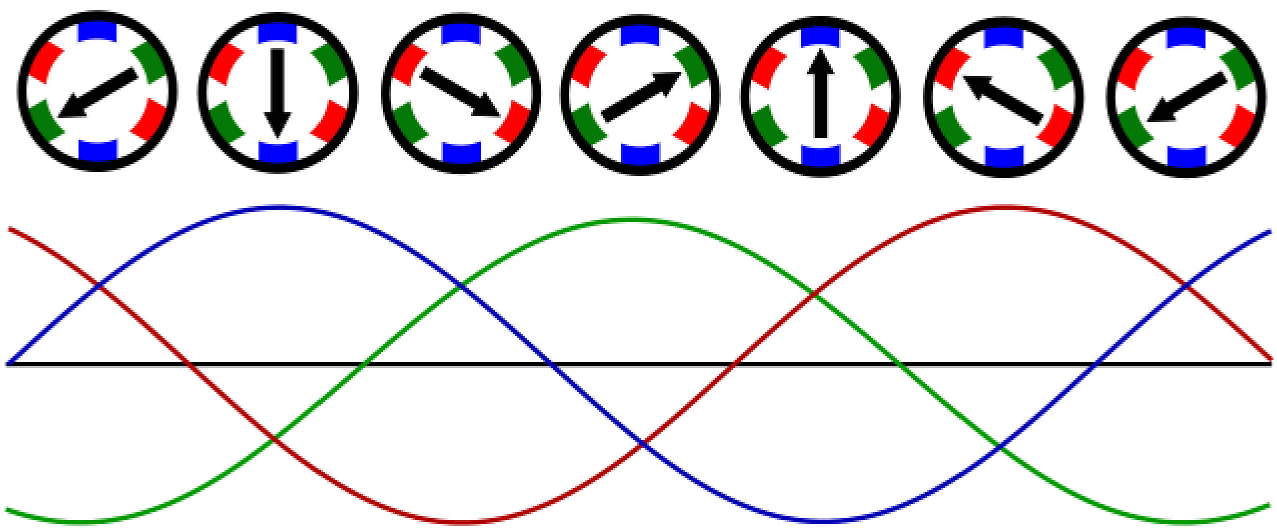
### Asynchronous Induction Motors vs. Synchronous Motors

90% of the energy consumed by motors is consumed by induction motors, which have rotor rotation that is close to but ultimately *asynchronous* with respect to airgap flux rotation speed. However, the terminology can get confusing. Asynchronous induction motors are the primary focus of the textbook and this webpage pertaining to electrical energy systems. It is important however to clearly distinguish these from synchronous motors (see below).

### Stators and Rotating Magnetic Fields

A 3-phase motor *stator* is itself stationary (as the name implies) but it produces a rotating air-gap field via its field windings which acts on the rotor inducing it to turn.

In the diagram that follows,[270] the 3-phase motor has 3 pairs of windings (green, red, and blue), one pair for each separate phase, and separated by 120˚. The 3 sine waves shown are the air-gap fluxes produced by each of the paired windings. For instance, the magnetic field produced by the blue winding oscillates in a plane between these top and bottom blue windings, going from maximal amplitude pointing to the top winding, passing through zero magnitude, and continuing to maximal amplitude pointing to the bottom winding. These add together in vector form to yield the resultant (phasor sum) magnetic air-gap flux vector depicted as the rotating black arrow:



An animated GIF here[271] shows how the 3 separate windings produce magnetic vector components that sum to yield a rotating net magnetic field vector (RMF).

For 3-phase stators, the magnitude of the rotating total flux vector is constant and equal to 1.5 $\varphi_{max}$, where $\varphi_{max}$ is the peak flux magnitude generated by a single winding. The textbook shows an example of a clockwise

[269] *EEAI3* p. 395-480
[270] https://en.wikipedia.org/wiki/Rotating_magnetic_field
[271] https://en.wikipedia.org/wiki/Synchronous_motor#Starting_methods
Unfortunately, Word document cannot show captured animated GIFs

rotating total magnetic field vector, i.e., it maintains a constant magnitude but a continuously rotating direction.[272]

The direction of magnetic flux for any winding is determined by the right hand rule, with conventional current flow direction represented by direction the curving fingers point, and magnetic field represented by the thumb.

## Airgap Flux Rotation Speed

The total airgap flux vector completes a revolution with each AC cycle, or $\tau = 1/f = 1/60$ second, at least for a "two-pole" arrangement. A single phase stator with only one pair of windings has 2 poles. Typically f is 60 Hz in the US (50 Hz in many other countries). In general, the *synchronous speed* $n_s$ is the rotational speed of the airgap flux that is determined by the frequency of the supply voltage, and is given by

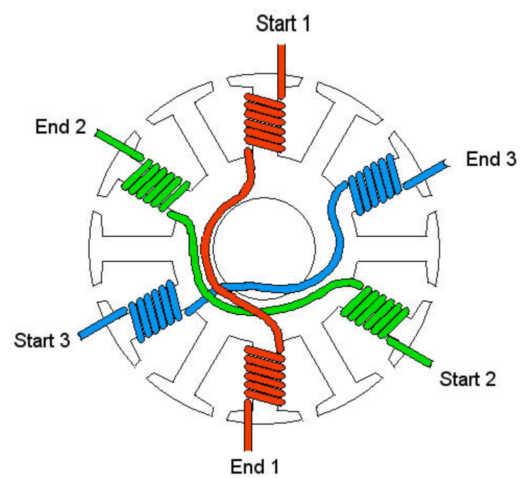
$n_s = 2f/p$ (in rev/s) = $120f/p$ (in rpm)

where p is the number of stator poles (p is twice the number of pole pair windings pp).

A two-pole machine is one in which each phase has two opposing windings (i.e., "poles", in which one winding creates a N pole when the opposite creates a S pole, etc.) Here, the synchronous speed $n_s$ is given by

$n_s = 2f/p$ (in rev/s) = 60 rev/s = 3600 rpm

The diagram[273] to the right illustrates a 2-pole 3-phase machine stator, having 6 windings (poles). For this 2-pole motor, the synchronous speed $n_s$ is also given by




$n_s = 2f/p$ (in rev/s) = 60 rev/s = 3600 rpm

A four-pole 3-phase machine has 2x3 = 6 pairs of windings, or 12 windings (poles). For this 4-pole motor, the synchronous speed $n_s$ is given by

$n_s = 2f/p$ (in rev/s) = 30 rev/s = 1800 rpm

A 10-pole machine operated at 60 Hz has a synchronous speed given by

$n_s = 2f/p$ = 12 rev/s = 720 rpm.

## Induced Voltage, Current, Torque, and Slip of Rotor of Asynchronous AC Induction Motors

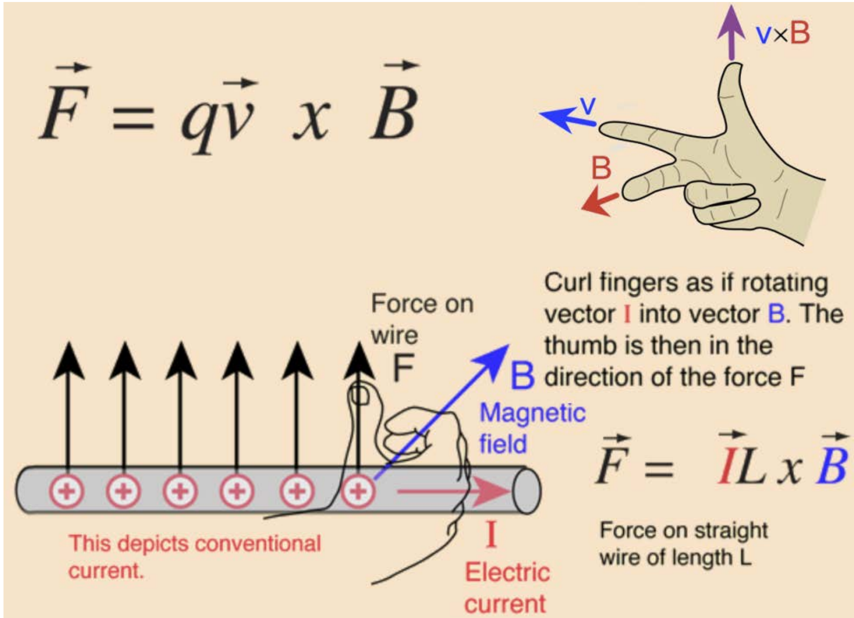
The rotating airgap flux is the driving force of the motor's *rotor* rotation. The rotating magnetic field **B** induces a voltage e in the rotor conductors per *Faraday's Law*.[274] This leads to a rotor current **i** which interacts with **B** to cause a force and thus a torque **T** on the rotor per the Lorentz equation (Laplace Force):[275]

The top equation and diagram depicts the Lorentz force on a positively charged particle. The diagram shows how the right hand rule applies: the velocity vector **v** of a positive charge (index finger) crossed (anti-commutatively) with the magnetic field vector **B** (along middle finger) yields the Lorentz force vector **F** (along thumb).

The bottom equation is the Lorentz force adapted to a positive charge current **I** flowing in a wire of length L in a magnetic field **B**. The lower diagram depicts again how the right hand rule applies: the velocity vector **v** has become the positively charged current vector **I** (direction of palm of hand); the magnetic field vector **B** (along

[272] *EEAI3* p. 396-398
[273] http://www.southernsoaringclub.org.za/a-BM-motors-3.html
[274] http://hyperphysics.phy-astr.gsu.edu/hbase/electric/farlaw.html
[275] *EEAI3* p. 403

fingers curling in from palm) yields the Lorentz force vector **F** (along thumb), which becomes a torque in N-m on the rotor.[276]




However, the rotor's rotation typically lags somewhat behind the airgap flux rotations (due to friction and load), so that the actual rotor and shaft rotation is somewhat less than $n_s$. Such motors may be termed *asynchronous ac induction motors*.

The quantity *slip* defines the extent by which the rotor's rotation rate n differs from the airgap flux's rotation rate $n_s$:

$$S = (n_s - n) / n_s$$

or in terms of angular velocity,

$$S = (\omega_s - \omega) / \omega_s$$

where $\omega = (2\pi/60)\, n$ [units of ω are rad/s, n is in rpm]. At startup when ω = 0, S = 1. Slip is close to 0 when the load is minimal and the rotor is rotating at almost the synchronous speed. A typical value of S is less than 0.1 or 10%, often 1% to 4% slip.[277]

## Types of Induction Motors

A wide variety of induction motors exists, and I can only touch on some of the key aspects. Most of the large motors used in industry are three-phase induction motors. "Single-phase motors are used almost exclusively to operate home appliances such as air conditioners, refrigerators, well pumps, and fans. They are generally designed to operate on 120 V or 240 V. They range in size from fractional horsepower to several horsepower, depending on the application."[278] (The reference source just cited also discusses multispeed motors and several types of synchronous motors—Holtz motors and Warren motors—as well as universal motors, which are series wound motors that can operate on DC or AC).

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[276] Diagrams adapted from https://upload.wikimedia.org/wikipedia/commons/thumb/d/d2/Right_hand_rule_cross_product.svg/2000px-Right_hand_rule_cross_product.svg.png and http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magfor.html

[277] *EEAI3* p. 402

[278] http://www.industrial-electronics.com/electric_prin_2e_19.html

## Motor nameplate specifications

Motor nameplate specifications are summarized here,[279] and may include some of the following (emphasis on NEMA specifications):

- **Manufacturer's Type**
- **Rated Volts** (at which the motor is designed to operate optimally)
- **Full Load Amps FLA** (when full-load torque and HP are reached)
- **Rated Frequency** (operating Hz)
- **Number of Phases** (1 or 3)
- **Full Load RPM** "RPM" (when voltage and frequency are at rated values)
- **Synchronous Speed** (of the rotating airgap magnetic field, not the same as the actual rotor rotation speed)
- **Insulation Class** (the 20,000 hr. life temperature, ranging from 105 °C to 180 °C, expressed as A, B, F, or H)
- **Maximum Ambient Temperature AMB**
- **Altitude** (max. height above sea level at which motor will remain within its design temperature—
  motors are less efficiently cooled when air is thin)
- **Time Rating** (length of time the motor can operate at its rated load and max. ambient temp—standard motors are rated for continuous use, specialized motors such as a pre-lube motor allow shorter duration use)
- **Horsepower HP** (rated shaft horsepower at rated speed) and closely related **Torque**, for which
  HP = speed [RPM] × Torque [lb-ft] / 5250); or
  HP = speed [RPM] × Torque [lb-in] / 63025);
- **Locked Rotor kVA Code** (starting inrush current expressed as code letters ABCDEFGHJKLMNPR
  based on kVA/HP ratios ranging in value from 0 to 15.99, where the value increases from A to R)
- **Power Factor PF** (ratio of active power W to apparent power VA, expressed as a %)
- **Service Factor SF** (factor of overloading the motor can handle for short periods when operating
  within correct voltage tolerances—
  e.g., a 10 HP motor with SF 1.15 can operate at 11.5 HP for short periods)
- **Full Load Nominal Efficiency** (average power output / power input for this motor model,
  given as a %, ideal is 100%)
- **Frame Size** (standard motor dimensions, expressed as a size number and letter designation)
- **NEMA Design Letter** for Induction Motors (A, B, C, or D)
- **Enclosure Types ENCL** (include ODP, TEFC, TENV, TEAO, TEWD, EXPL, HAZ including various classes)
- **Thermal Protection** (temp. sensing and protection techniques, including:
  None
  Auto = Auto shutoff w auto reset
  Man = Auto shutoff with manual reset
  Impedance
  Thermostat T-St which open and close contacts)
- **Other aspects:**
  Bearings
  Shaft Type
  Mounting
  Power Factor Correction (i.e., capacitor size)
  Special Markings and Certifications (UL, CSA, ASD)

## Rotor Design

The rotor may consist of windings that are shorted internally and permanently as part of the rotor structure (termed a *squirrel cage* rotor), or externally through a system of *slip rings* and *brushes*. In a 3-phase slip ring rotor, each phase has an isolated slip ring and circuit (the slip ring is in continuous contact with its brush), and the 3 circuits are connected in Wye configuration. (DC motors, not otherwise discussed here, often make use of *split ring commutators* and brushes that periodically reverse the current direction in the rotor circuit.[280]) Squirrel cage rotors lack the flexibility of varying speed and torque, in contrast to wound rotors with accessible rotor circuits.

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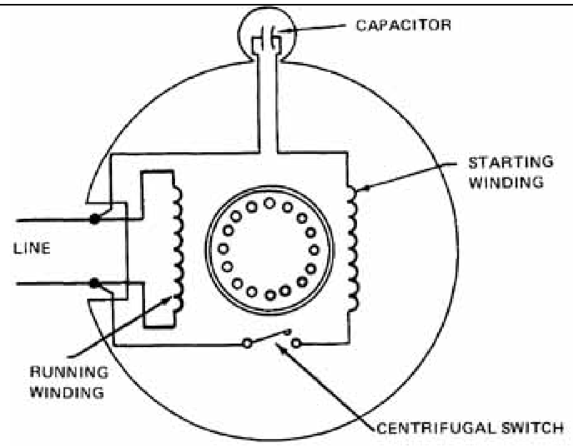
279 http://www.pdhonline.org/courses/e156/e156content.pdf
280 hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motdc.html

For squirrel cage internally shorted rotors (illustrated here[281]), "The motor rotor shape is a cylinder mounted on a shaft. Internally it contains longitudinal conductive bars (usually made of aluminum or copper) set into grooves and connected at both ends by shorting rings.. The name is derived from the similarity between this rings-and-bars winding and a squirrel cage. The solid core of the rotor is built with [insulated] stacks of electrical steel laminations [in which the conductive bars are embedded]… The rotor has a smaller number of slots than the stator and must be a non-integer multiple of stator slots so as to prevent magnetic interlocking of rotor and stator teeth at the starting instant… The field windings in the stator of an induction motor set up a rotating magnetic field through the rotor. The relative motion between this field and the rotor induces electric current in the conductive bars. In turn these currents lengthwise in the conductors react with the magnetic field of the motor to produce force acting at a tangent orthogonal to the rotor, resulting in torque to turn the shaft. In effect the rotor is carried around with the magnetic field but at a slightly slower rate of rotation. The difference in speed is called *slip* and increases with load… The conductors are often *skewed slightly* along the length of the rotor to reduce noise and smooth out torque fluctuations that might result at some speeds due to interactions with the pole pieces of the stator. The number of bars on the squirrel cage determines to what extent the induced currents are fed back to the stator coils and hence the current through them…"[282]

## Startup and Rotor Rotation Direction

The presence of and direction of rotation of a 3-phase induction motor is determined and assured by the rotating airgap flux. For single phase induction motors, special efforts must be taken to assure that the rotor rotation will begin on startup in the correct direction and with adequate torque. Various techniques include the following:




Circuit diagram of capacitor-start induction motor[283]

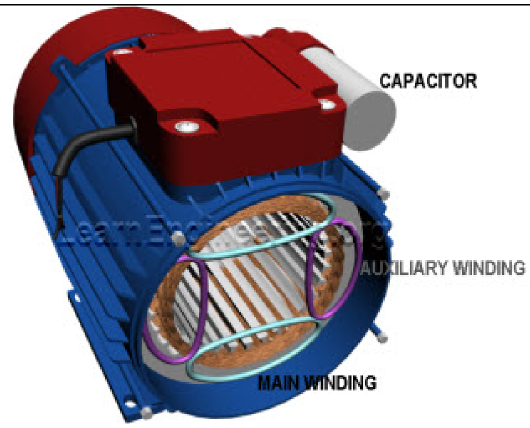
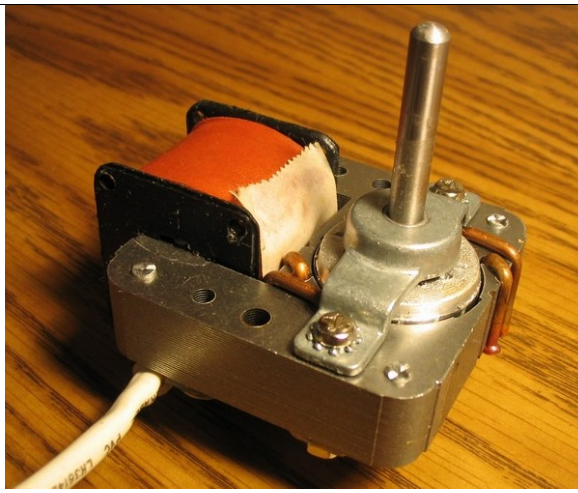
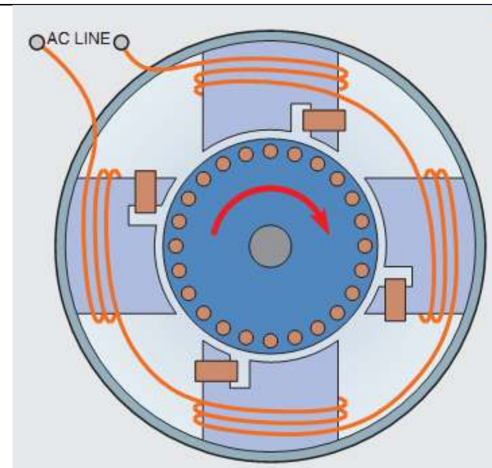



Diagram of auxiliary stator windings with capacitor-start induction motor.[284] Centrifugal switch turns off aux winding once the motor attains a certain speed.

[281] https://www.youtube.com/watch?v=LtJoJBUSe28
[282] https://en.wikipedia.org/wiki/Squirrel-cage_rotor
[283] http://www.industrial-electronics.com/elecy4_22.html diagram
[284] http://www.learnengineering.org/2013/08/single-phase-induction-motor.html diagram



Shaded pole motor.[285] The direction of rotation is from the unshaded side to the shaded (ring) side of the pole—thus, this motor rotor will rotate in the clockwise direction.




Single phase 4-pole shaded-pole induction motor.[286] The shading coil opposes a change of flux as current increases or decreases

(1) **Resistance-start split-phase induction-run motor**. "The resistance-start induction-run motor is so named because the out-of-phase condition between start and run winding current is caused by the start winding being more resistive than the run winding... The start winding ... does have some inductive reactance, preventing the current from being in phase with the applied voltage. Therefore, a phase angle difference of 35° to 40° is produced between these two currents, resulting in a rather poor starting torque."[287]

(2) **Auxiliary stator windings with capacitor**, which operates initially as a 2-phase motor.

This approach may take the form of a **permanent-split capacitor PSC motor (capacitor-start capacitor-run)**, which is somewhat inefficient but works adequately in smaller motors up to about 1/4 hp. The split refers to the division of current between the run winding and the start winding.

A **capacitor-start induction motor (capacitor-start induction-run motor.)** is more efficient if a larger capacitor is used to start a single phase induction motor via the auxiliary winding, and this winding is switched out by a centrifugal switch once the motor is running at perhaps 75% of rated speed. The auxiliary winding may consist of many more turns of heavier wire than used in a resistance split-phase motor to mitigate excessive temperature rise. The result is that more starting torque is available for heavy loads like conveyers and air conditioning compressors.[288,289] "When a capacitor of the proper size is connected in series with the start winding, it causes the start winding current to lead the applied voltage. This leading current produces a 90° phase shift between run winding current and start winding current. Maximum starting torque is developed at this point."[290]

The switching off of the starting winding can also be accomplished by a relay when a centrifugal switch is unsuitable: "When [motors] are hermetically sealed, a centrifugal switch cannot be used to disconnect the start winding. A device that can be mounted externally is needed to disconnect the start windings from the circuit. Starting relays perform this function. There are three basic types of starting relays used with the resistance-start and capacitor-start motors: 1. Hot-wire relay; 2. Current relay; 3. Solid state starting relay."[291]

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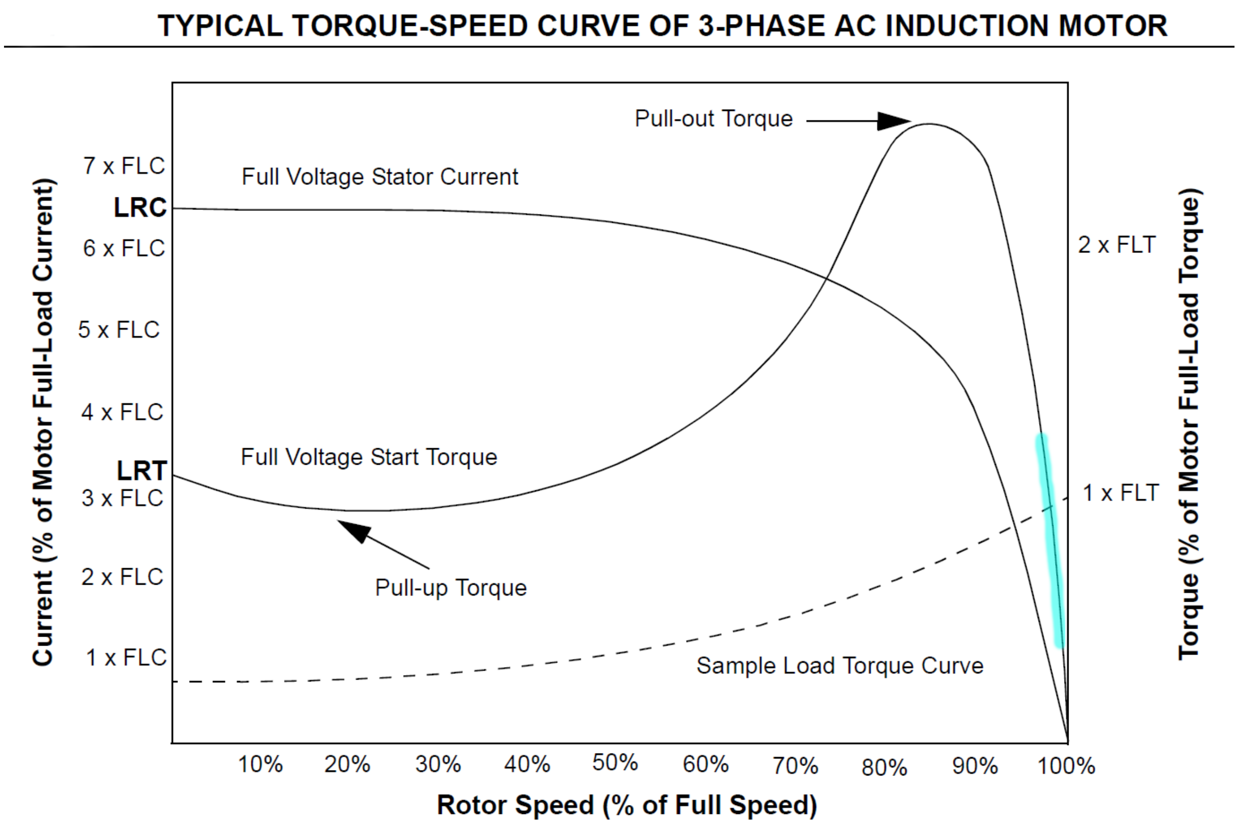
[285] https://en.wikipedia.org/wiki/Shaded-pole_motor image
[286] http://www.industrial-electronics.com/electric_prin_2e_19.html diagram
[287] http://www.industrial-electronics.com/electric_prin_2e_19.html
[288] https://www.youtube.com/watch?v=awrUxv7B-a8
[289] http://www.allaboutcircuits.com/textbook/alternating-current/chpt-13/single-phase-induction-motors/ including image of capacitor-start induction motor.
[290] http://www.industrial-electronics.com/electric_prin_2e_19.html
[291] http://www.industrial-electronics.com/electric_prin_2e_19.html

(3) **Shaded pole motor**: These provide low starting torque, low cost, low efficiency, no capacitors, and no start switch. They are used on small direct drive fans, etc. This is "basically a small squirrel-cage motor in which the auxiliary winding is composed of a copper ring or bar surrounding a portion of each pole... This auxiliary single-turn winding is called a shading coil. Currents induced in this coil by the magnetic field create a second electrical phase by delaying the phase of magnetic flux change for that pole (a shaded pole) enough to provide a 2-phase rotating magnetic field. The direction of rotation is from the unshaded side to the shaded (ring) side of the pole... Since the phase angle between the shaded and unshaded sections is small, shaded pole motors produce only a small starting torque relative to torque at full speed... The common, asymmetrical form of these motors ... has only one winding, with no capacitor or starting windings/starting switch, making them economical and reliable... Because their starting torque is low, they are best suited to driving fans or other loads that are easily started. They may have multiple taps near one electrical end of the winding, which provides variable speed and power via selection of one tap at a time, as in ceiling fans. Moreover, they are compatible with TRIAC-based variable-speed controls, which often are used with fans. They are built in power sizes up to about 1⁄4 horsepower (190 W) output. Above 1⁄3 horsepower (250 W), they are not common, and for larger motors, other designs offer better characteristics."[292] "[Due to the shaded pole,] The magnetic field would be seen to rotate across the face of the pole piece."[293]

Torque vs. Speed Relationship for Induction Motors




In the diagram above,[294] the Torque, Current, and Rotor Speed are graphed. The rotor speed % is the % of the *synchronous speed* $n_s$. An asynchronous induction motor always rotates at a speed less than 100% of $n_s$.

> STARTING CHARACTERISTIC: "Induction motors, at rest, appear just like a short circuited transformer and if connected to the full supply voltage, draw a very high current known as the '***Locked Rotor Current***' or LRC. They also produce torque which is known as the '***Locked***

[292] https://en.wikipedia.org/wiki/Shaded-pole_motor
[293] http://www.industrial-electronics.com/electric_prin_2e_19.html
[294] http://www.t-es-t.hu/download/microchip/an887a.pdf , both diagram and quoted text. Also discussed in *EEAI3* p. 410-1

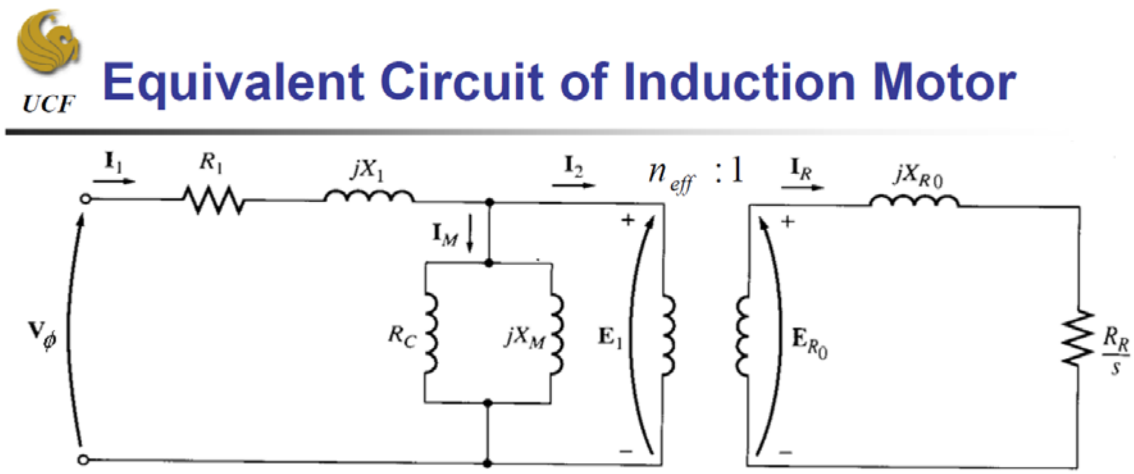
***Rotor Torque***' or LRT. The Locked Rotor Torque (LRT) and the Locked Rotor Current (LRC) are a function of the terminal voltage of the motor and the motor design. The LRC of a motor can range from 500% of ***Full-Load Current*** (FLC) to as high as 1400% of FLC. Typically, good motors fall in the range of 550% to 750% of FLC. The starting torque of an induction motor starting with a fixed voltage will drop a little to the minimum torque, known as the ***pull-up torque***, as the motor accelerates and then rises to a maximum torque, known as the ***breakdown torque*** or ***pull-out torque*** [a region to be avoided for sustained operation], at almost full speed and then drop to zero at the **synchronous speed** [$n_s$]. The curve of the start torque against the rotor speed is dependent on the terminal voltage and the rotor design. The LRT of an induction motor can vary from as low as 60% of [***Full Load Torque***] FLT to as high as 350% of FLT. The pull-up torque can be as low as 40% of FLT and the breakdown torque can be as high as 350% of FLT. Typically, LRTs for medium to large motors are in the order of 120% of FLT to 280% of FLT. The [***Power Factor***] PF of the motor at start is typically 0.1-0.25 [indicating the current drawn by the motor is primarily the magnetizing current and is almost purely inductive], rising to a maximum as the motor accelerates and then falling again as the motor approaches full speed.

RUNNING CHARACTERISTIC: Once the motor is up to speed, it operates at a low slip, at a speed determined by the number of the stator poles. [It is operating at nearly constant speed in the nearly steady-state region where developed torque = rated load torque FLT, a region that I have marked in color.] Typically, the full-load slip for the squirrel cage induction motor is less than 5%. The actual full-load slip of a particular motor is dependent on the motor design. The typical base speed of the four pole induction motor varies between 1420 and 1480 RPM at 50 Hz, while the synchronous speed is 1500 RPM at 50 Hz. The current drawn by the induction motor has two components: ***reactive component (magnetizing current)*** and ***active component (working current)***. The magnetizing current is independent of the load but is dependent on the design of the stator and the stator voltage. The actual magnetizing current of the induction motor can vary, from as low as 20% of FLC for the large two pole machine, to as high as 60% for the small eight pole machine. The working current of the motor is directly proportional to the load… A typical medium sized four pole machine has a magnetizing current of about 33% of FLC. A low magnetizing current indicates a low iron loss, while a high magnetizing current indicates an increase in iron loss and a resultant reduction in the operating efficiency. Typically, the operating efficiency of the induction motor is highest at 3/4 load and varies from less than 60% for small low-speed motors to greater than 92% for large high-speed motors. The operating PF and efficiencies are generally quoted on the motor data sheets…"

## Equivalent Circuit for Induction Motors and Power Analysis

As with transformers, analysis of reactance and currents etc. can be aided by the use of equivalent circuit approximations, which refer the rotor circuit to the stator circuit using the turns ratio. This is detailed in the textbook (p. 406-7), and a representative sample from another source is shown here:[295]

[295] http://www.eecs.ucf.edu/~tomwu/course/eel4205/notes/21%2520IM%2520Equivalent%2520Circuits.pdf




According to the textbook, this analysis allows determination of a load resistance or developed power resistance $R_d$, which is the electrical representation of the mechanical load. In the power analysis, the input power $P_{in}$ is divided into Stator copper loss $P_{cu1}$ (wire resistance), Stator core loss $P_c$ (eddy currents, etc.) and Airgap power $P_g$. Airgap power $P_g$ is divided into Rotor copper loss $P_{cu2}$ and Developed power $P_d$. Developed power $P_d$ is divided into Rotational losses $P_{rotation}$ and Output power $P_{out}$, the latter being the final goal.

The motor efficiency $\eta = P_{out} / P_{in}$

## Synchronous Motors

Most *synchronous* electric machines are dual action electromechanical converters: when converting rotational motion to electricity they are called *generators*, and when converting electricity into rotational motion they are called *motors*.[296] "Generator action will be observed if the [rotor] field poles are 'driven ahead of the resultant air-gap flux by the forward motion of the prime mover' [such as a water turbine]. Motor action will be observed if the [rotor] field poles are 'dragged behind the resultant air-gap flux by the retarding torque of a shaft load'."[297]

The rotor may consist of a rare earth permanent magnet (such as samarium-cobalt), giving such a motor a high power to volume ratio, or it may be excited by an external separate DC source. The magnetic field of the rotor aligns itself with the rotating magnetic field of the stator and the rotor thus spins at the synchronous speed.

The following quotes pertain to synchronous motors.[298]

"A synchronous electric motor is an AC motor in which, at steady state, the rotation of the shaft is synchronized with the frequency of the supply current; the rotation period is exactly equal to an integral number of AC cycles. Synchronous motors… have a rotor with *permanent magnets or electromagnets that turns in step with the stator field at the same rate…* The synchronous motor does not rely on current induction to produce the rotor's magnetic field. Small synchronous motors are used in timing applications such as in synchronous clocks, timers in appliances, tape recorders and precision servomechanisms in which the motor must operate at a precise speed; speed accuracy is that of the power line frequency, which is carefully controlled in large interconnected grid systems… Synchronous motors are available in sub-fractional self-excited sizes [up to] to high-horsepower industrial sizes. In the fractional horsepower range,

[296] *EEAI3* p. 395
[297] https://en.wikipedia.org/wiki/Synchronous_motor
[298] https://www.youtube.com/watch?v=Vk2jDXxZIhs good animation

most synchronous motors are used where precise constant speed is required.  These machines are commonly used in analog electric clocks, timers and other devices where correct time is required."[299]

"... Synchronous motors are very rarely used below 40kW output because of the higher cost compared to induction motors.  In addition to the higher initial cost synchronous motors need a DC excitation source and starting and control devices are usually more expensive...  Where applications involve high kW output and low speed synchronous motors are economical compared to induction motors.  The various classes of service for which synchronous motors are employed may be classified as:  Power factor correction; Voltage regulation; Constant speed constant load drives.  Applications include the following:[300]

> • Synchronous motors are used in generating stations and in substations connected to the busbars to improve the power factor...  These machines when over excited delivers the reactive power to grid and helps to improve the power factor of the system.  The reactive power delivered by the synchronous motors can be adjusted by varying the field excitation of the motor...
> • Because of the higher efficiency compared to induction motors they can be employed for loads which require constant speeds.  Some of the typical applications of high speed synchronous motors are such drives as fans, blowers, dc generators, line shafts, centrifugal pumps, compressors, reciprocating pumps, rubber and paper mills
> • Synchronous motors are used to regulate the voltage at the end of transmission lines
> • In textile and paper industries synchronous motors are employed to attain wide range of speeds with variable frequency drive system"

A commercial description from WEG.net of industrial synchronous motors includes the following:[301]

> "Why Using Synchronous Motors?
>
> The application of synchronous motors in industry most often results in considerable economic and operational advantages caused by their performance characteristics. The main advantages are:
>
> Power Factor Correction:
>
> Synchronous motors can help to reduce electric energy costs and to improve the efficiency of the power system by supplying reactive energy to the grid they are connected...
>
> Constant Speed:
>
> Synchronous motors are capable of maintaining constant speed operation under overload conditions and/or during voltage variations, observing the limits of maximum torque (pull-out).
>
> High Torque Capacity:
>
> Synchronous motors are designed with high overload capability, maintaining constant speed even in applications with great load variations.
>
> High Efficiency:
>
> Synchronous motors are designed to provide high efficiency under a large range of operational conditions providing significant savings with energy costs along its lifetime.
>
> Greater Stability in the Operation with Frequency Inverters:
>
> Synchronous motors can operate in a wide speed range, while maintaining stability regardless of load variation (e.g.: rolling mill, plastic extruder, among others)."

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[299] https://en.wikipedia.org/wiki/Synchronous_motor
See also *EEAI3* p. 451-457
[300] http://electricalquestionsguide.blogspot.com/2012/09/applications-synchronous-motors.html
[301] http://www.weg.net/files/products/WEG-specification-of-electric-motors-50039409-manual-english.pdf

### Other Motors

I have omitted much discussion of DC motors as well as Linear Induction Motors and Stepper Motors. The latter are used to precisely position the rotor in steps, as with hard disk drives, printers, plotters, scanners, fax machines, medical equipment laser guidance systems, robots, and actuators.

## Synchronous Generators

Time does not permit adequate coverage of this interesting topic. The stator is similar to that of the induction motor, and typically consists of 3-phase windings in multiple pole configurations. The rotor is excited by an external DC source through a slip ring system. The rotor is spun by the prime mover (water turbine, steam turbine, wind turbine, etc.)

By way of example, the 32 main generators at the Three Gorges dam each weigh about 6,000 tonnes and are designed to produce more than 700 MW of power each. The designed head [pressure] of the generator is 80.6 meters (264 ft). Three Gorges uses Francis turbines. Turbine diameter is 9.7/10.4 m (VGS design/Alstom's design) and rotation speed is 75 revolutions per minute. Rated power is 778 MVA, with a maximum of 840 MVA and a power factor of 0.9. The generator produces electrical power at 20 kV. The stator, the biggest of its kind, is 3.1/3.0 m in height. Average efficiency is over 94%, and reaches 96.5%.[302]

Recall that a 10-pole machine operated at 60 Hz has a synchronous speed given by

$$n_s = 2f/p = 2 * 60 \text{ Hz}/ 10 \text{ poles} = 12 \text{ rev/s}$$

Assuming 50 Hz power generation, the Three Gorges rotation rate of 75 RPM predicts the following number of poles (in the rotor, as it turns out!):

$$p = 2f/n_s = 2 * 50 \text{ Hz}/ (75/60 \text{ [rev/s]}) = 80 \text{ poles}$$ [303]

[302] https://en.wikipedia.org/wiki/Three_Gorges_Dam and http://www.chinatourmap.com/three_gorges_dam/power_generation.html

[303] http://www.slideshare.net/endutesfa/three-gorges-project Slide 46 of 81

# Electrical Safety

This is an important topic, and some of what I present below may have mistakes. Please consult primary sources before making any important decisions regarding electrical safety. This discussion is partly drawn from the textbook Chapter 9. I have omitted the topic of biological and health effects of low frequency magnetic fields.[304]

## Human Electrical Shock Physiology

The capacity of humans to withstand electrical shock was studied by Charles F. Dalziel in the 1950's and 1960's. He found that human responses correlate more closely with current rather than voltage. The following table draws on his published results for average responses (but individuals may vary!):[305]

| | *Bodily effect* | *Gender* | *DC* | *60 Hz AC* | *10 kHz AC* |
|---|---|---|---|---|---|
| % affected not stated | Slight sensation at point(s) of contact | Men | 1 mA | 0.4 mA | 7 mA |
| | | Women | 0.6 mA | 0.3 mA | 5 mA |
| | Threshold of bodily perception | Men | 5.2 mA | 1.1 mA | 12 mA |
| | | Women | 3.5 mA | 0.7 mA | 8 mA |
| 50%, but 0.5% lose control | Pain, with voluntary muscle control maintained | Men | 62 mA | 9 mA | 55 mA |
| | | Women | 41 mA | 6 mA | 37 mA |
| 50% of pop. | Pain, with loss of voluntary muscle control | Men | 76 mA | 16 mA | 75 mA |
| | | Women | 51 mA | 10.5 mA | 50 mA |
| | Severe pain, difficulty breathing | Men | 90 mA | 23 mA | 94 mA |
| | | Women | 60 mA | 15 mA | 63 mA |
| ~0.5% of pop. | Possible heart fibrillation after three seconds | Men | 500 mA | 100 mA | |
| | | Women | 500 mA | 100 mA | |

Here, the *let-go* current (the threshold level above which the victim *cannot let go* of the conductor), is shown as "pain, with loss of voluntary muscle control". The stage specifying difficulty breathing is also called *respiratory tetanus*, because respiratory muscles are in spasm and breathing is impaired or impossible. *Cardiac fibrillation*, for which the research was conducted in non-humans, occurs at 100 mA in 0.5% of men and at 67 mA in 0.5% of women (according to the textbook). In general, Dalziel found that women are more sensitive than men to electrical current, and that AC is more hazardous than DC. *Secondary shock current* describes currents that are possibly painful, but do not cause permanent tissue injury or death. *Primary shock current* can exceed let-go threshold, cause heating and burns (including of nerves), respiratory tetanus, or ventricular fibrillation.

The OSHA gives a similar table (to right), based on another researcher's work: "This table shows the general relationship between the amount of [probably AC] current received and the reaction when current flows from the hand to the foot for just 1 second."[306]

**Effects of Electric Current in the Human Body**

| Current | Reaction |
|---|---|
| Below 1 milliampere | Generally not perceptible |
| 1 milliampere | Faint tingle |
| 5 milliamperes | Slight shock felt; not painful but disturbing. Average individual can let go. Strong involuntary reactions can lead to other injuries. |
| 6–25 milliamperes (women) | Painful shock, loss of muscular control* |
| 9–30 milliamperes (men) | The freezing current or "let-go" range.* Individual cannot let go, but can be thrown away from the circuit if extensor muscles are stimulated. |
| 50–150 milliamperes | Extreme pain, respiratory arrest, severe muscular contractions. Death is possible. |
| 1,000–4,300 milliamperes | Rhythmic pumping action of the heart ceases. Muscular contraction and nerve damage occur; death likely. |
| 10,000 milliamperes | Cardiac arrest, severe burns; death probable |

* If the extensor muscles are excited by the shock, the person may be thrown away from the power source.

The severity of electrical shock depends on the following:[307]

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[304] *EEAI3* p. 310

[305] http://www.ibiblio.org/kuphaldt/socratic/output/shock.pdf,
image annotated MCM based on Charles F. Dalziel, "Deleterious Effects of Electric Shock", 1961 available at http://www.electriciancalculators.com/dalziel/dalziel_study.pdf. Also, *EEAI3* p. 274-5

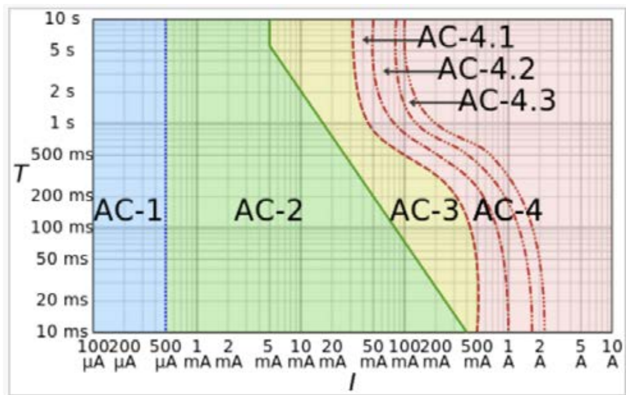
[306] W.B. Kouwenhoven, "Human Safety and Electric Shock," Electrical Safety Practices, Monograph, 112, Instrument Society of America, p. 93. November 1968, cited in https://www.osha.gov/Publications/osha3075.pdf OSHA, 2002.

[307] *EEAI3* p. 275

- The amount of *current* (amps) flowing through the body. Current is determined partly by *voltage* and partly by the *resistance* in the current path.
- *Voltage*: Higher voltage may be more likely to throw the victim away from the hazard, due to violent muscle contraction, but there is no guarantee, and otherwise higher current will result from higher voltage.
- *Body Resistance*: is higher in bones, fat, and tendons, lower in nerves, blood vessels, and muscles. Total resistance hand to hand varies widely, from 13,500 Ω for driest skin conditions to 820 Ω for the wettest conditions (e.g., sweaty palms). The textbook *EEAI3* suggests rule of thumb resistances of 500 Ω for each hand + arm, 500 Ω for each leg, and 100 Ω for the torso, with a useful average of 1000 Ω between two hands or two feet.
- The current's *path* through the body. For instance, in dogs a path from one forelimb to one hindlimb is more likely to cause ventricular fibrillation than one passing between the two forelimbs (i.e., ECG leads II or III vs. lead I, apparently because the former passes more current through the heart).[308]
- The length of time (*duration*) the body remains part of the circuit. Dalziel estimated the time to ventricular fibrillation is $t = (K/I)^2$, where K is 116 for < 70kg body weight and 154 for > 70 kg. (See also graph below regarding duration and VF.)
- The current's *frequency*. The let-go current follows a U-shaped pattern, being lowest in the 50 to 100 Hz range, and much higher for DC and for AC of 1000 Hz and above.[309]
- *Ground resistance* and other factors affecting resistance. E.g., standing on an insulating substance is much safer than standing in water (see below).

The graph to the right depicts in greater detail the risk of ventricular fibrillation for AC current passing from left hand to both feet, from exposure in specified mA and duration ranges.[310]

The graph below, from the excellent article on Electrical Safety by Walter H. Olson, shows the effect of frequency per Charles Dalziel.[311]




Log-log graph of the effect of alternating current *I* of duration *T* passing from left hand to feet as defined in IEC publication 60479-1.[18]

AC-1: imperceptible

AC-2: perceptible but no muscle reaction

AC-3: muscle contraction with reversible effects

AC-4: possible irreversible effects

AC-4.1: up to 5% probability of ventricular fibrillation

AC-4.2: 5-50% probability of fibrillation

AC-4.3: over 50% probability of fibrillation

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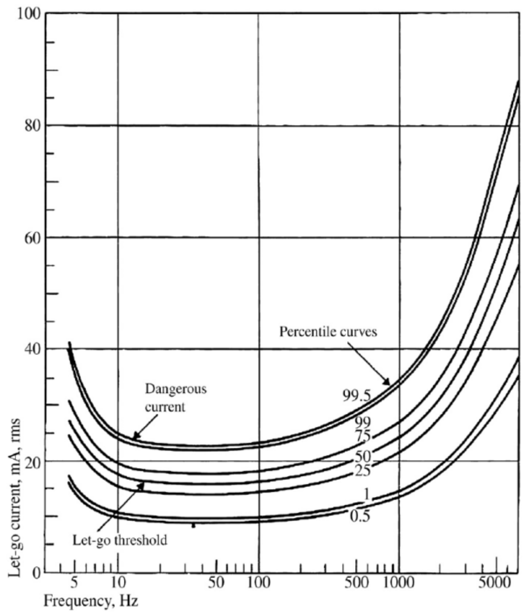
308 L. A. Geddes, et al, “Threshold 60-Hz Current Required for Ventricular Fibrillation in Subjects of Various Body Weights”, *IEEE Transactions On Biomedical Engineering*, November 1973. Dr. Geddes was a much respected mentor for me at Baylor College of Medicine in 1965-1966.

309 *EEAI3* p. 278

310 Weineng Wang, Zhiqiang Wang, Xiao Peng, “Effects of the Earth Current Frequency and Distortion on Residual Current Devices”, *Scientific Journal of Control Engineering*, Dec 2013, Vol 3 Issue 6 pp 417-422, article in Chinese thus unreadable by me, cited in https://en.wikipedia.org/wiki/Electric_shock

311 https://eva.fing.edu.uy/pluginfile.php/68296/mod_resource/content/1/c14.pdf




**Let-go current versus frequency** Percentile values indicate variability of let-go current among individuals. Let-go currents for women are about two-thirds the values for men. (Reproduced, with permission, from C. F. Dalziel, "Electric Shock," *Advances in Biomedical Engineering*, edited by J. H. U. Brown and J. F. Dickson III, 1973, 3, 223–248.)

## Ground Resistance, Ground Potential, Ground Potential Rise, Touch and Step Potentials

These are important considerations in evaluating shock risk.

Voltages are measured between 2 points. The center of the Earth is taken to have potential or voltage of 0V. This is the point of reference by which other voltages may be compared. "The real Earth … is electrically neutral. This means that it has the same number of electrons and protons, so their charges cancel out overall. Scientifically, we describe this by saying that the Earth has an Electric Potential of zero."[312] Our textbook *EEAI3* specifically defines this condition as existing at the center of the Earth. This allows for the possibility that there may be a potential (a so-called *Ground Potential* GP) between a point on or near the Earth's surface (such as on water pipes, building or substation foundations, transmission tower footings, etc.), and the center of the Earth. This GP is relevant when high voltage towers leak current to the ground, or lightning strikes nearby. In these cases, the local ground potential may differ from 0V and create a hazard.

We may determine the effective resistance, the *Ground Resistance* GR, between the ground point of interest and Earth's center. This resistance determines how much current flows between the ground object having nonzero voltage potential and the zero voltage at the Earth center.

Mathematical computation of GR is easiest for a buried conducting hemisphere located at the Earth's surface. For current I flowing through this hemisphere of radius $r_0$, current density J for a point on or outside its surface at a distance r from its center ($r \geq r_0$) is given by:[313]

$$J = \frac{I}{2\pi r^2}$$

We may compute the voltage (GP) between points on concentric virtual hemispheric equipotential surfaces found on or outside the buried hemispheric conductor. If these points are on concentric virtual hemispheres of radius $r_a$ and $r_b$, where $r_0 < r_a < r_b$, the voltage between the virtual hemispheres (and thus any two points, one on each) is

$$V_{ab} = \frac{\rho I}{2\pi}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

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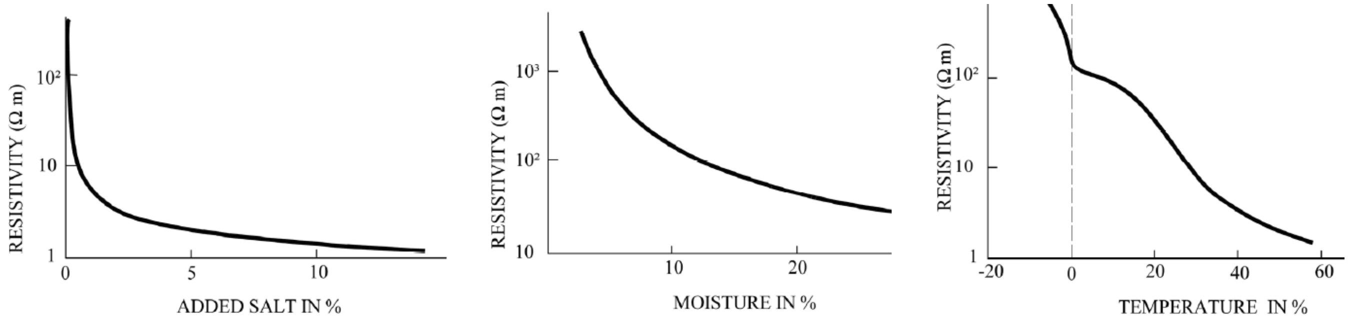
312 https://www.st-andrews.ac.uk/~www_pa/Scots_Guide/info/earth/earth.htm
313 *EEAI3* p. 281

where ρ is the electrical resistivity of soil in the region (assumed to be uniform), measured in Ω-m. Values of ρ can vary widely, due to varying soil types, moisture content, salt or chemical content, and temperature [for instance, frozen conditions raise resistivity].[314] Values of ρ range from 10 Ω-m for wet organic soil to 10,000 Ω-m for bedrock.[315] Some of the factors affecting resistivity are demonstrated in this diagram adapted from IEEE:[316]




**Earth resistivity variations (Rudenberg [B55]):**
**(a) salt, (b) moisture, and (c) temperature**

An extensive commercial webpage describes current methodologies (like the Wenner 4-point [four-pin] Soil Resistivity Test) for measuring soil resistivity.[317] The textbook *EEAI3* illustrates a simpler three-point test.

**Hemisphere**: The Ground Resistance $R_g$ between a point on the buried hemisphere (at $r_0$) and the center of the Earth is estimated by

$$R_g = \frac{\rho}{2\pi}\left[\frac{1}{r_0} - \frac{1}{r_{\text{center of earth}}}\right] = \frac{\rho}{2\pi r_0}$$

**Rod**: For a conducting grounding rod buried to depth l and having radius r, the IEEE quoting Thug gives the Ground Resistance $R_g$ estimated by two different approximations, the first also given in the textbook:[318]

$$R_g = \frac{\rho}{2\pi l}\ln\left[\frac{2l+r}{r}\right] \quad \text{or} \quad R_g = \frac{\rho}{2\pi l}\ln\left[\frac{4l}{r} - 1\right]$$

**Circular Plate**: A circular plate of radius r at the surface has approximate GR:

$$R_g = \frac{\rho}{4r}$$

Ground resistance is relevant when a person or other living thing is standing near a source of ground potential. If each foot has a Ground Resistance $R_f$, which varies by soil and shoe sole conditions, etc., the combined (parallel) Ground Resistance for both feet is 0.5 $R_f$.

## Touch potential

If a transmission tower has a leakage or fault current I (diagram below[319]) passing down to ground (perhaps from a dirty, salt-laden, wet, or otherwise faulty insulator), a point on the structure of the tower where a man touches it will have a *Ground Potential Rise* GPR, i.e., the voltage above 0V, given by

$$GPR = I_{tg} R_g$$

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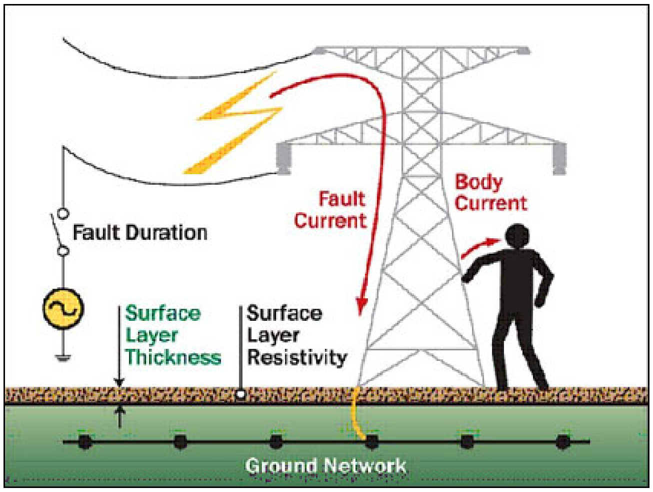
314 https://en.wikipedia.org/wiki/Soil_resistivity
315 Determining The Soil Resistivity To Design a Good Substation Grounding System: http://electrical-engineering-portal.com/determining-the-soil-resistivity-to-design-a-good-substation-grounding-system
See also http://www.transcat.com/media/pdf/App-Ground-SoilResistivity.pdf for effect of soil type, moisture, and T.
316 IEEE Guide for Measuring Earth Resistivity, Ground Impedance, and Earth Surface Potentials of a Grounding System", IEEE Std 81-2012
317 http://www.esgroundingsolutions.com/what-is-soil-resistivity-testing/
318 IEEE Guide for Measuring Earth Resistivity, Ground Impedance, and Earth Surface Potentials of a Grounding System", IEEE Std 81-2012; also *EEAI3* p. 283.
319 http://www.esgroundingsolutions.com/what-is-step-and-touch-potential/

where $I_{tg}$ is the tower to ground fault current, and $R_g$ is the Ground Resistance at that point on the tower. The man touching the tower is exposed to this *Touch Potential*, and a part of the $I_{tg}$ current becomes diverted through his body and both feet into the ground. The current through the man is modeled in the textbook (p. 288) based on his presenting a resistance ($R_{man}$ + 0.5 $R_f$ ), which is in parallel to the Ground Resistance $R_g$ of the tower.




For example, if the fault current down the tower is 10A into moist soil with ρ = 100 Ω-m (grounded at a hypothetical hemisphere having r=0.5 m), the man's resistance $R_{man}$ is 1000 Ω, and each foot has sole area = 0.02 m², then

$R_g$ = 32 Ω        V at touch point (GPR) = $IR_g$ = 320V

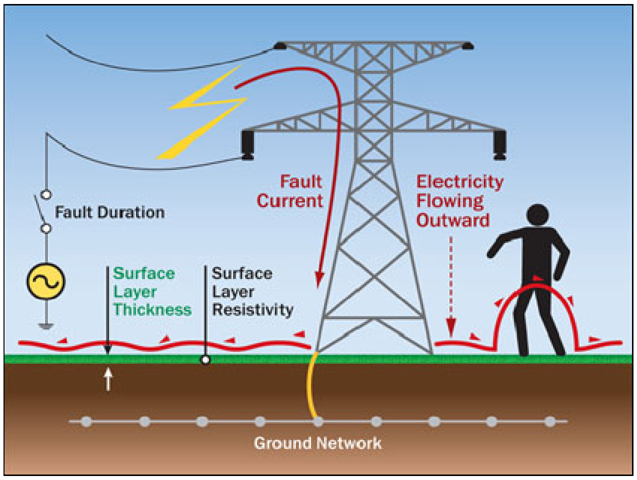
$R_f$ = 3ρ = 300 Ω (each foot)        $I_{man}$ = 270.7 mA        $V_{man} = I_{man}R_{man}$ = 270 V

This level of current will kill the man in less than a second of contact.

## Step potential

This is another hazard of being near a source (leaking transmission tower, lightning bolt, etc.) passing a large current into and along the ground. If a current is passing horizontally, the soil surface is not equipotential. Instead, there is a gradient of surface voltage (potential), highest near the source and tapering off with horizontal distance (as well as with vertical or oblique underground distance).

See diagram at right, from a website that also provides the following. "Hazardous Step Potentials or step voltage can occur a significant distance away from any given site. The more current that is pumped into the ground, the greater the hazard. Soil resistivity and layering plays a major role in how hazardous a fault occurring on a specific site may be. High soil resistivities tend to increase Step Potentials. A high resistivity top layer and low resistivity bottom layer tends to result in the highest step voltages close to the ground electrode: the low resistivity bottom layer draws more current out of the electrode through the high resistivity layer, resulting in large voltage drops near the electrode. Further from the ground electrode, the worst case scenario occurs when the soil has conductive top layers and resistive bottom layers: in this case, the fault current remains in the conductive top layer for much greater distances away from the electrode."[320]




With step potentials, there is a difference in potential arising from a ground current passing below the feet. The feet are at different voltages, unless they happen to be at exactly the same distance from the source, the soil and foot resistances are identical, etc.)

The textbook *EEAI3* gives an example as follows:

The fault current down the tower is 1000A into moist soil (ρ = 100 Ω-m) and is grounded at a hemisphere (for which r is not stated). The man's leg to body resistance $R_{man}$ is 1000 Ω. Each foot a and b has Ground Resistance $R_f$. A voltage difference $V_{ab}$ exists between the feet due to the current, higher for foot placed at a which is closer to the hemisphere than foot placed at b. The stride length (distance between feet on the ground) is about 0.6 m.

[320] http://www.esgroundingsolutions.com/what-is-step-and-touch-potential/

The solution applies Thévenin's theorem[321] to compute an equivalent Thévenin resistance $R_{th}$ and Thévenin voltage $V_{th}$. Here, $V_{th}$ is simply the voltage $V_{ab}$ when the feet are off the ground (no human load across a and b). $R_{th}$ is simply the sum of the two $R_f$ for the open circuit, or $2R_f$.

$$V_{th} = V_{ab} = \frac{\rho I}{2\pi}\left[\frac{1}{r_a} - \frac{1}{r_b}\right] = \frac{100\times 100}{2\pi}\left[\frac{1}{5} - \frac{1}{5.6}\right] = 341\text{V}$$

$R_f = 3\rho = 300\ \Omega$ (each foot)

$$I_{man} = \frac{V_{th}}{R_{th}+R_{man}} = \frac{V_{th}}{2R_f+R_{man}} = \frac{341}{600+1000} = 213.13\text{ mA} \quad \text{(hazardous, quite possibly lethal)}$$

The step voltage is $V_{step} = I_{man} \times R_{man} = 213.13\text{V}$

If a 1 m grounding rod is used rather than a hemisphere, $I_{man}$ = 308.75 mA, even worse. In either case, The victim may fall over from the initial shock, then have lethal current passing through heart etc. .

In summary, it can be dangerous to stand or walk near a high current leak passing along the ground.

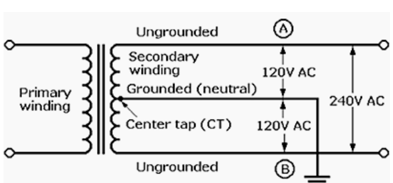
## Home Electrical Safety

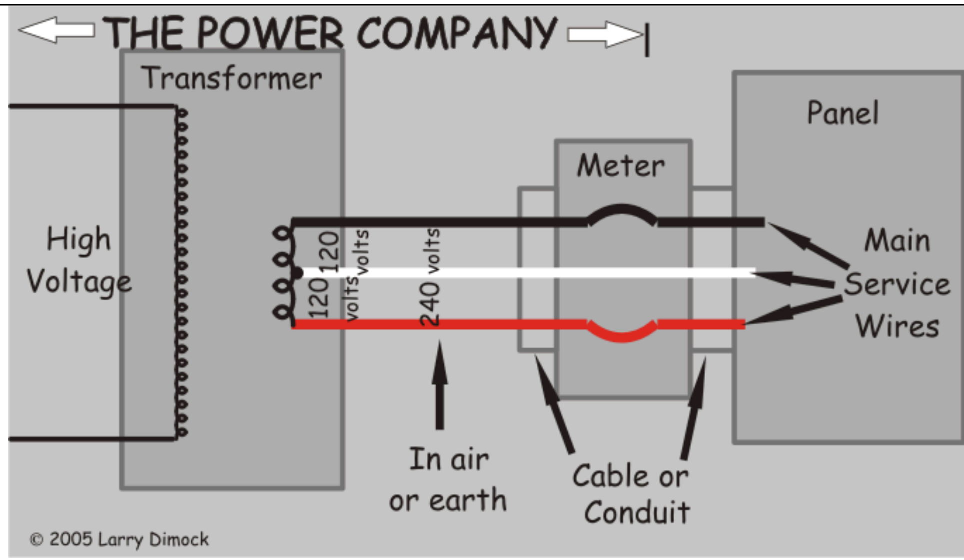
The following focuses on US and North American residential standards, and mostly ignores 3-phase.

### Home Electrical Service

Home electrical service enters the home like this:[322]

Distribution Transformer providing L-N-L **single phase** 120/240V service (aka split phase) (above). Diagram (below)[323]



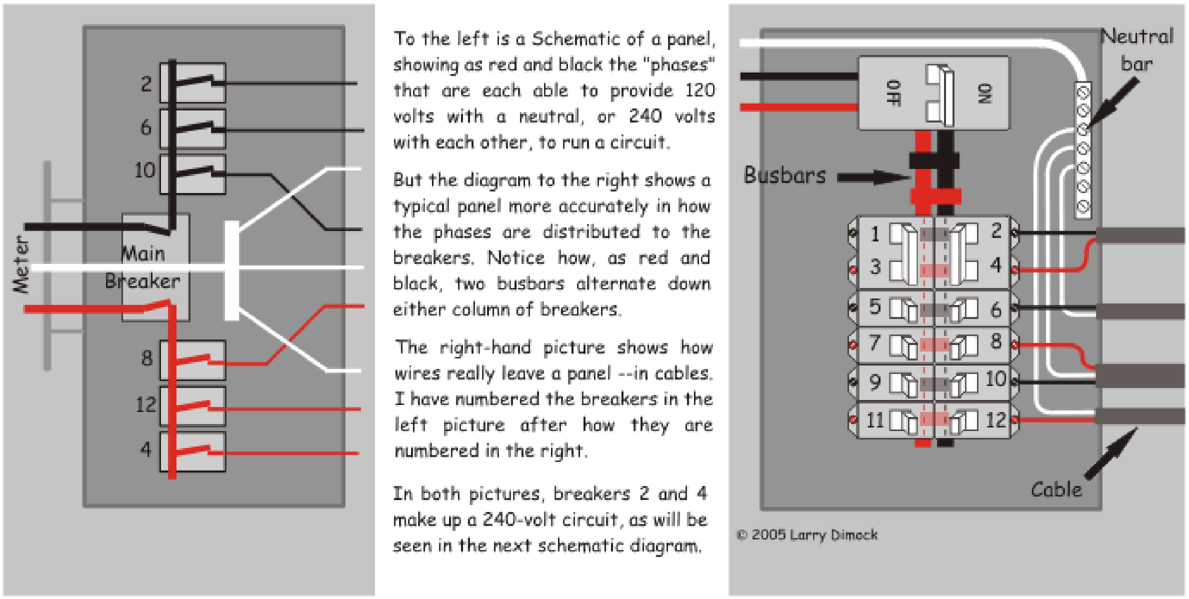

**Single Phase**: The distribution transformer secondary (on the left above) provides one of the 3 possible phases to the home, specifically a *hot* black line and a *hot* red line (synonym: *service wires*, *phases, legs*). This level of service is called *single phase* or *split phase*. With single phase (split phase), there is 240 V rms between the lines (the *line to line* L-L voltage).

Single phase also provides a white (or gray) *neutral conductor* (synonym: *grounded conductor*, *neutral point*) arising from the transformer secondary as a center tap. There is 120V rms between either phase and the neutral conductor, the *Line-to-Neutral* or L-N voltage. (The neutral conductor arises from the center of the wye connection in a 3-phase power system.)

321 https://en.wikipedia.org/wiki/Th%C3%A9venin's_theorem (I have not mastered this technique)
322 http://www.thecircuitdetective.com/bkgrd.htm
323 http://electrical-engineering-portal.com/power-distribution-configurations-with-three-three-phase-hot-power-lines (diagram)

(For 3-Phase power, there are "three ac voltages separated from each other by 120 electrical degrees, or by a third of a cycle. These systems deliver power over three hot wires where the voltage across any two hot wires measures 208 V [rms]."[324])

After the meter, the lines power feeds into the *breaker panel* (synonyms: *load center*, *distribution board*, *junction box*, *service panel*, or *circuit panel*) wherein are found the circuit breakers (aka *miniature circuit breakers* MCBs). "The United States electrical codes require that the neutral be connected to earth at the 'service panel' only and at no other point within the building wiring system… The neutral is neither switched nor fused."[325]




Ideally and typically, there should be a *main breaker*, which is a 2-pole breaker, typically 200A, that can shut off all power being fed to the busbars (bus bars) that in turn feed the branch circuit breakers. These can be manually tripped or automatically trip when current exceeds 200A in either phase.

I have not made a careful study of the rather complex technology of circuit breakers. Home circuit breakers typically have air gap circuit interruption (use air alone to extinguish the arc) and trip after a nominal current has been exceeded for a specified time delay. 240 circuits use 2-pole circuit breakers to trip both phases simultaneously if either exceeds its trip amperage. (Three phase circuits must use three-pole common trip breakers.) I read that circuit breakers should be turned off and on to make sure they are not stuck and that they are in good operating condition. There are provisos however, see this article[326].

The figure that follows is excerpted from a specification sheet for a GE 40A miniature circuit breaker (MCB).[327] It is a log-log plot. On the horizontal axis is plotted Amperes, and on the vertical axis is plotted t (time to clear in seconds). The green band represents the range of currents and time delays for which the circuit breaker *clears* (opens). It is apparent that as overload current increases (in this case, currents exceeding 40 A), the time delay before the MCB trips (clears) becomes shorter. When the current barely exceeds 40A, it

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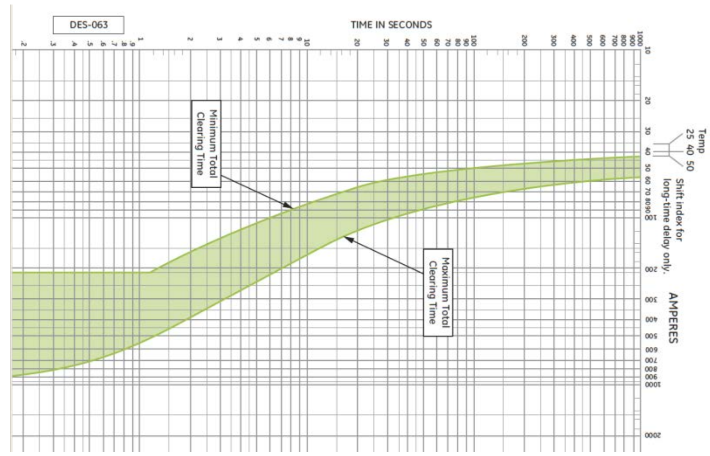
324 http://www.belden.com/blog/datacenters/3-Phase-Power-Wye-It-Matters.cfm;
see also https://www.dataforth.com/catalog/pdf/an110.pdf , which shows that all line-to-line phasor supply voltages are line-to-neutral voltages multiplied by $\sqrt{3} = 120\sqrt{3} = 120 \cdot 1.732 = 207.8$V rms. A standard 4-wire 3-phase wye system with line-to-neutral voltages of 120 volts and V1N chosen as the reference phasor at zero degrees has line-to-line voltages of: $V12 = 208\angle 30°$ $V23 = 208\angle -90°$ $V31 = 208\angle 150°$

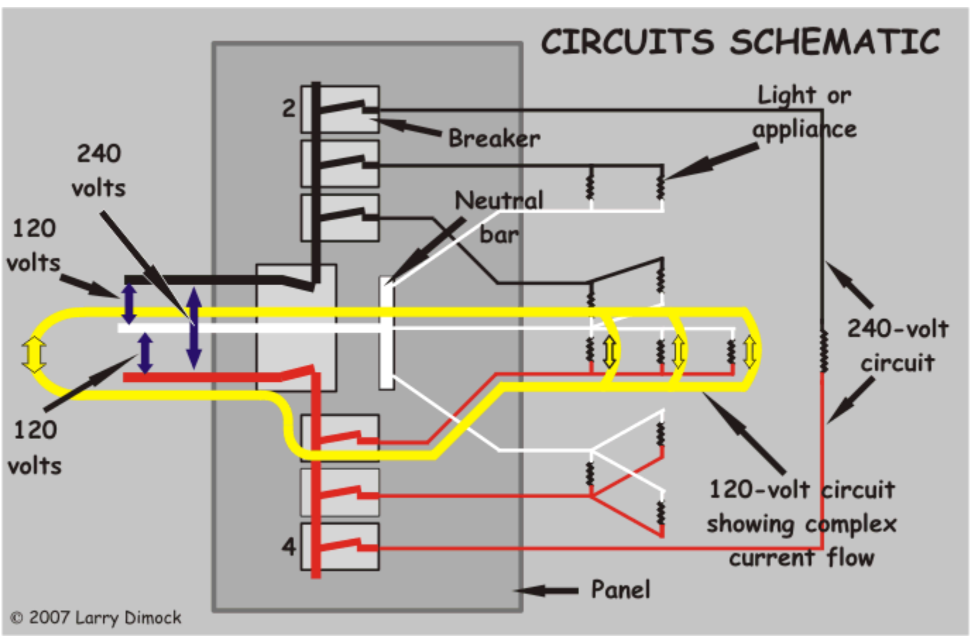
325 https://en.wikipedia.org/wiki/Electrical_wiring_in_North_America

326 http://www.examiner.com/article/basic-home-electricity-part-1-how-to-maintain-circuit-breakers-gfcis-and-afcis

327 Sheet DES-063. Molded Case Circuit Breakers Q Line Type THQB/THHQB - Model C. Graph "Depicts the Long-time Delay and Instantaneous Time-current Curves. Fetch this sheet from http://www.geindustrial.com/your-business/consulting-design-engineer/time-current-curves

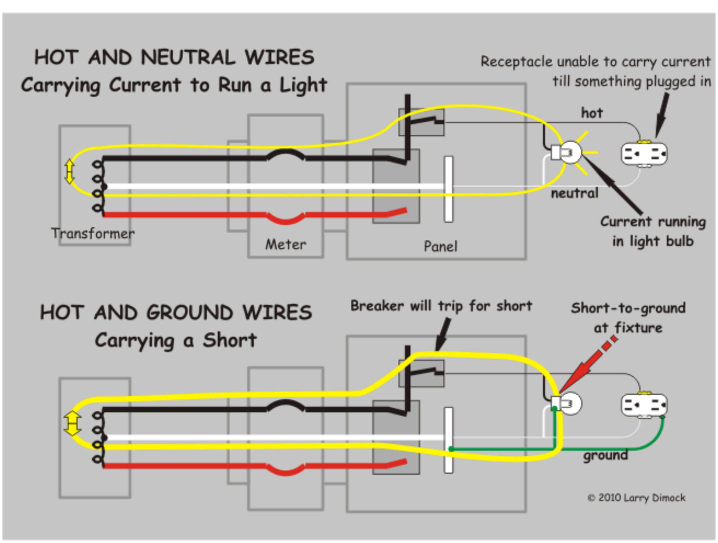
takes many seconds to trip (apparently as many as 1000 s). For currents in the 150 A range, the MCB trips in about 2 to 10 seconds. For higher currents exceeding 200 A, the MCB clears in 0.01 to 1 second. For extreme currents > 1000 A, the graph shows clearing in less than 0.02 seconds. However, at some excess short circuit current, the MCB may not be able to properly function and might be destroyed—this is not shown in the graph. It is also apparent that this MCB clears at a lower current for lower temperatures (~35A at 25° F) whereas it clears at a higher current for higher temperatures (~42A at 50° F), though this is a modest difference.




The diagram to the left illustrates how 240V and 120V *circuits* flow to and from loads, and often follow complex paths involving *sub-branches* (yellow). 120V current flows from red or black to neutral, whereas 240V current flows from one hot to the other.

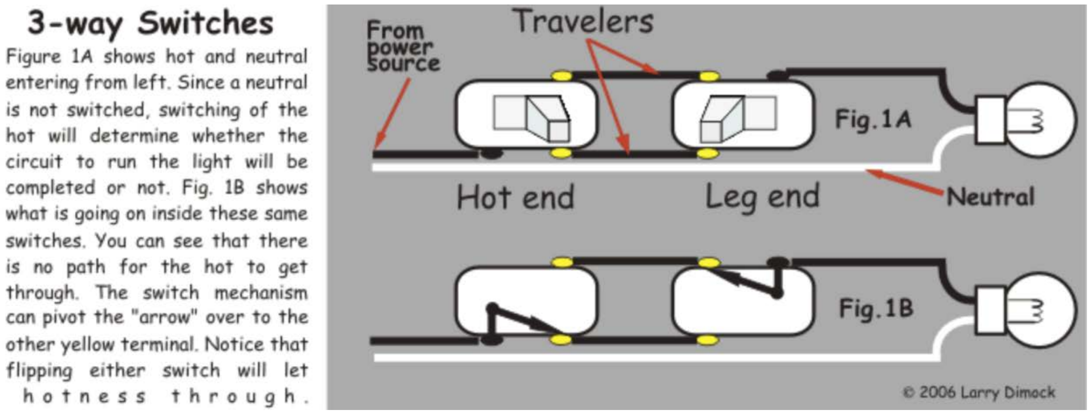
"By Code, a dedicated circuit is used for each of most large appliances like the electric range, electric water heater, air conditioner, or electric dryer; these as well as electric heaters will have two (joined) breakers in order to use 240 volts rather than the 120 volts used by most other items. A dedicated circuit of 120 volts is usually provided for each dishwasher, disposal, gas or oil furnace, and clothes washer. Most other 120-volt circuits tend to serve a number (from 2 to 20) of lights and plug-in outlets. There are usually two circuits for the outlets in the kitchen/dining area, and these use a heavier wire capable of 20 amps of flow…"




"Besides black, red, and white wires, the cables in homes wired since the 1960's also contain a bare or green 'ground(ing)' wire. [diagram to left] Like the neutral, it is ultimately connected to the transformer's grounded terminal, but this wire is not connected so as to be part of the normal path of flow around the circuit. Instead, it is there to connect to the metal parts of lights and appliances, so that a path is provided 'to ground' if a hot wire should contact such parts; otherwise you or I could be the best available path… When a ground wire does carry current, it is taking care of an otherwise dangerous situation; in fact, it usually carries so much flow suddenly, that it causes the breaker of the circuit to trip…"

"Even when a circuit is switched off, we call hot wires hot to remind ourselves of their future potential to shock and to distinguish them from neutrals and grounds.  Only hot wires should be switched, never neutrals or grounds."[328]

Finally, to clear up one of the great mysteries of home life, I have included this explanation of 3-way light switches:

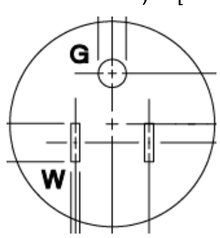



Note that the diagram shows the light off.  To switch the light on, either switch may be flipped (causing the internal arrow to shift from top pointing to bottom pointing or conversely), completing a hot line connection to the bulb.[329]

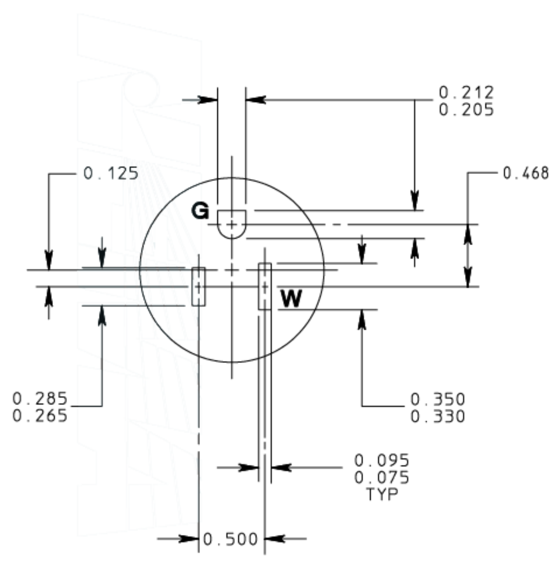
### Power Receptacles (Outlets, Sockets, and Female Connectors) and Plugs (Male Connectors)

Standard sizing and specifications of wiring devices (plugs, receptacles, plates, etc.) are given by NEMA.[330] These specifications include single gang wallplates, duplex plates, devices, plugs (blades, prongs), receptacles (sockets), etc.

A typical diagram for a single female NEMA 5-15R receptacle is shown to the right (with corresponding mirror-image male plug at lower left).  [Here, 5 signifies 125V grounded 3-wire Hot-Neutral-Ground, 15R signifies 15 amp receptacle, as opposed to 15P for plug].  The taller (wider) blade socket is connected to the neutral white wire.  The shorter blade socket is for the hot "line" black wire, and the socket marked G is for the green ground connection.  The ground blade of the plug is longer to establish a ground connection before the hot connection is made.




PLUG




RECEPTACLE

"NEMA connectors are power plugs and receptacles used for AC mains electricity in North America and other countries that use the standards set by the US National Electrical Manufacturers Association.  NEMA wiring devices are made in current ratings from 15 to 60 amperes, with voltage ratings from 125 to 600 volts.  Different combinations of contact blade widths, shapes, orientation, and dimensions create non-interchangeable

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[328] http://www.thecircuitdetective.com/bkgrd.htm  This website is an excellent source of household electrical information
[329] http://www.thecircuitdetective.com/3and4wyinfo.htm
[330] https://www.nema.org/standards/pages/wiring-devices-dimensional-specifications.aspx?#download (registration required)

connectors that are unique for each combination of voltage, electric current carrying capacity, and grounding system… NEMA 1 (two-prong, no DC safety ground) and NEMA 5 (three-prong, with safety ground pin) connectors are used for commonplace domestic electrical equipment; the others are for heavy duty or special purposes. NEMA 5–15R is the standard 15 amp capacity electric receptacle (outlet) found in the United States. Similar and interchangeable connectors are used in Canada and Mexico."[331]

**Locking Receptacles**: These permit curved blades to be rotated to lock into place by twisting (rotating) the plug. For example, L5-30 [locking] receptacles are common at marinas for secure boat connections. These consist of Hot-Neutral-Ground connections. More complex locking connectors include NEMA L14-50R, which has Hot-Hot-Neutral-Ground connections for either 125 or 240 V and up to 50A.

**Polarized plugs vs. Unpolarized plugs**: For polarized plugs having only 2 blades, the wider blade is for the neutral wire connection, whereas the narrower blade is for the line "hot" wire. "Polarized NEMA 1-15 plugs will not fit into unpolarized sockets, which possess only narrow slots. Polarized NEMA 1-15 plugs will fit NEMA 5-15 grounded sockets, which have a wider slot for the neutral blade. Some devices that do not distinguish between neutral and line, such as internally isolated AC adapters [and double insulated small appliances], are still produced with unpolarized narrow blades."[332]

**240V Plugs and Sockets**: These typically have a ground connector (for the green ground wire) and 2 hot/live identically sized connectors. Some of these have more than 3 connectors, for instance: "All NEMA 14 devices offer two hots, a neutral and a ground, allowing for both 120 V and 240 V."[333]

**Higher Voltage and 3-Phase Industrial Connectors**: Though not encountered in most homes, these are common in industry, hospitals, etc. For example, NEMA L23-50R is a locking receptacle for 50A 3-Phase connections in a range of voltages up to 600V, offering for the Wye 3-Phase configuration 3 hot connections (X, Y, and Z) plus Neutral and Ground.

**Color Coded Industrial Receptacles**: "…Although colors are not standardized by NEMA, some industries utilize colors for certain applications, following de facto standards:[334]

- A receptacle with a green dot is a so-called "hospital grade" device.
- A receptacle (any color) with an orange triangle, or an all-orange receptacle, is an isolated ground (IG) device, where the grounding pin of the receptacle is connected to ground independently of the frame of the receptacle and wiring outlet box.
- A blue receptacle may indicate built-in surge suppressors.
- A red receptacle may indicate a special-service outlet such as one connected to an emergency standby power source.
- At least one manufacturer makes a yellow receptacle, which identifies it as corrosion-resistant"

## Preventing Shock Hazards in the Home

### Earth Grounding versus Neutral Connections

Here are some observations from Wikipedia:

> "As the neutral point of an electrical supply system is often connected to earth ground, ground and neutral are closely related. Under certain conditions, a conductor used to connect to a system neutral is also used for grounding (earthing) of equipment and structures. Current carried on a grounding conductor can result in objectionable or dangerous voltages appearing on equipment enclosures, so the installation of grounding conductors and neutral conductors is carefully defined in electrical regulations…
>
> In North America, the cases of some kitchen stoves (ranges, ovens), cook tops, clothes dryers and other specifically listed appliances were grounded through their neutral wires as a

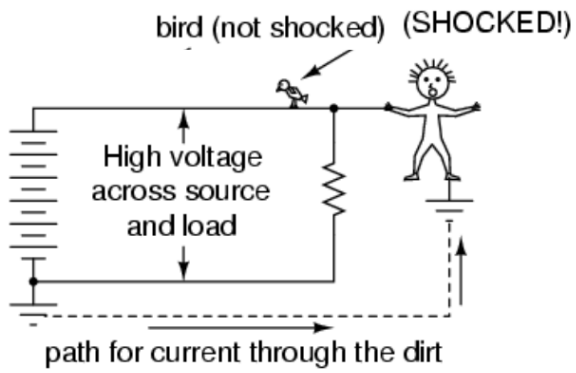
[331] https://en.wikipedia.org/wiki/NEMA_connector incl. diagrams and paraphrased text
[332] https://en.wikipedia.org/wiki/NEMA_connector
[333] ibid.
[334] ibid.

measure to conserve copper from copper cables during World War II. This practice was removed from the NEC [National Electrical Code] in the 1996 edition, but existing installations (called "old work") may still allow the cases of such listed appliances to be connected to the neutral conductor for grounding…

Combined neutral and ground conductors are commonly used in electricity supply companies' wiring and occasionally for fixed wiring in buildings and for some specialist applications where there is little alternative, such as railways and trams. Since normal circuit currents in the neutral conductor can lead to objectionable or dangerous differences between local earth potential and the neutral, and to protect against neutral breakages, special precautions such as frequent rodding down to earth (multiple ground rod connections), use of cables where the combined neutral and earth completely surrounds the phase conductor(s), and thicker than normal equipotential bonding must be considered to ensure the system is safe."[335]

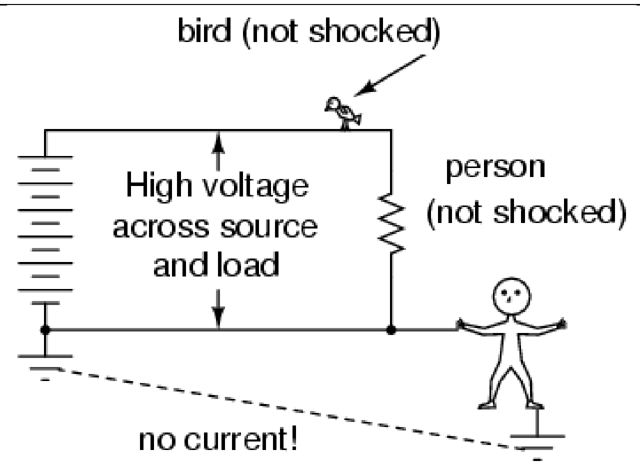
## Causes and Techniques for Reducing Shock Hazards

Ground connections are illustrated in diagrams below. DC voltages are typically depicted, but the principles generally apply to AC with minimal reinterpretation. Quotes, paraphrases, and diagrams are from here[336] unless otherwise noted.)



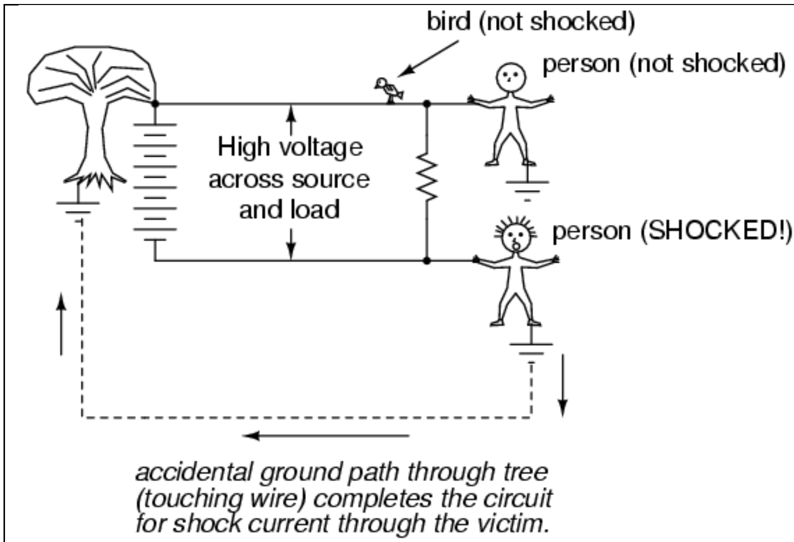

**Hazardous Hot Live Wire, Con**: Touching a live wire lets a standing grounded person be an alternate path to ground, and he is shocked. The bird on the wire is not in contact with the ground, thus is not shocked.

Circuit grounding ensures that at least one point in the circuit will be safe to touch (at the bottom of the load here, but not at the top, as shown to the right).

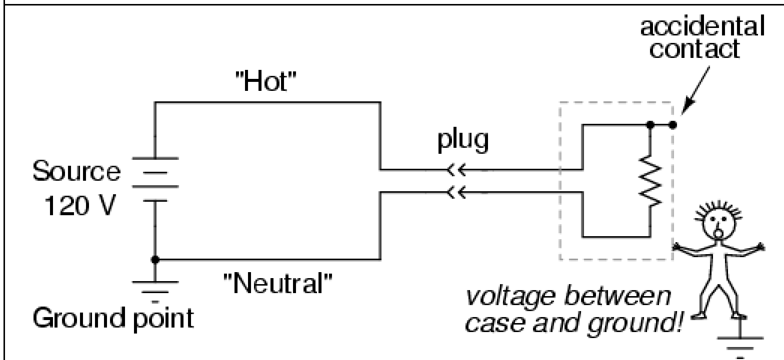



**Neutral Grounding, Pro**: By grounding one side of the load, the person is not shocked when touching that side. "Because the bottom side of the circuit is firmly connected to ground through the grounding point on the lower-left of the circuit, the lower conductor of the circuit is made electrically common with earth ground. Since there can be no voltage between electrically common points, there will be no voltage applied across the person contacting the lower wire, and they will not receive a shock. For the same reason, the wire connecting the circuit to the grounding rod/plates is usually left bare (no insulation), so that any metal object it brushes up against will similarly be electrically common with the earth."

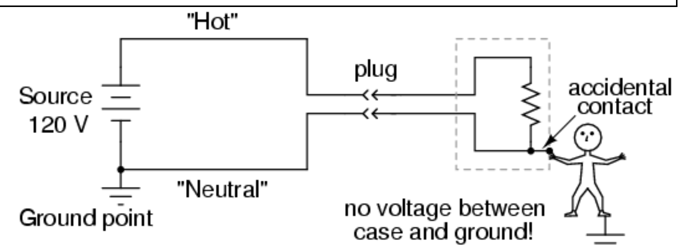
[335] https://en.wikipedia.org/wiki/Ground_and_neutral
[336] http://www.allaboutcircuits.com/textbook/direct-current/chpt-3/shock-current-path/



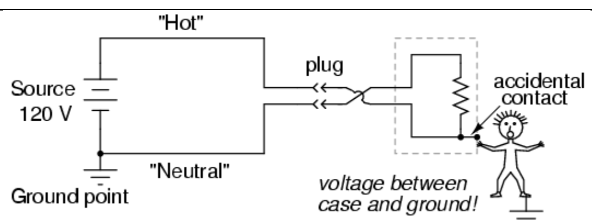

**Neutral Grounding Alone, Con**: If a circuit is completely ungrounded, a person touching just a single wire might seem safe. However, if part of the circuit becomes grounded through accident, such as a tree branch touching a hot power line, the person touching the same high voltage side becomes a path to ground that returns through the tree to the hot, and he is shocked.




**Floating Chassis, Con**: Inside the home, if an internal hot wire touches the metal external case of an ungrounded toaster (a so-called *floating chassis*), a person could be shocked by providing an alternate path to ground.



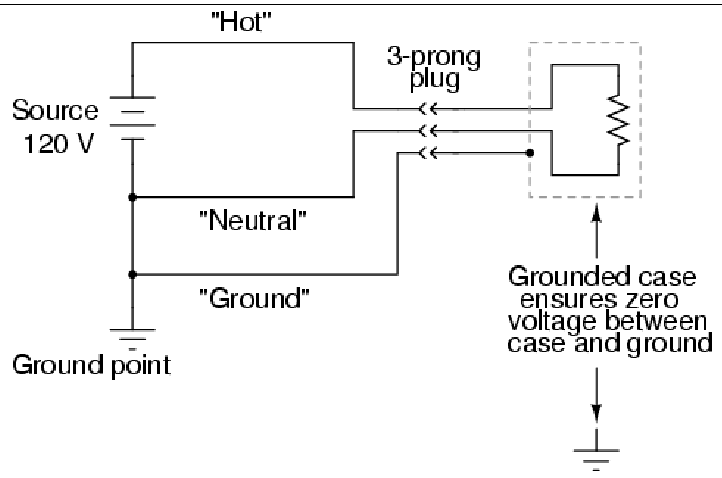

If the neutral wire contacts the case (floating chassis), there is no shock hazard. (Here, DC is shown, but the principle applies to AC as well.)




**Reversed Polarity, Con**: Designers try to assure that the hot wire inside the toaster will never contact the case. However, if the plug or receptacle is not polarized, and hot and neutral connections to the toaster are reversed, and there is an accidental internal hot wire connection to the case, the person might touch the hot case and be shocked. This is one of the major argument for polarization of plugs and receptacles.

It is important never to cut the third prong off a power plug when trying to fit it into a two-prong receptacle [or to use a 2-prong plug adapter]. If this is done, there will be no grounding of the appliance case to keep the user(s) safe…

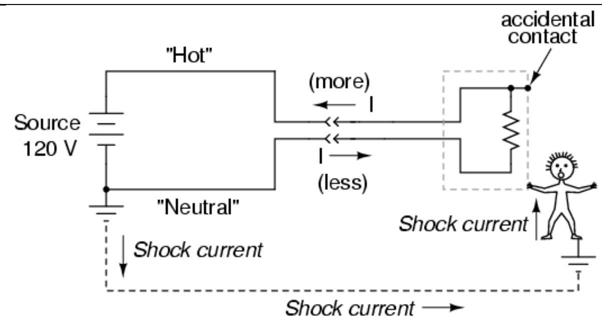
**Double Insulation, Pro**: Some engineers address the safety issue simply by making the outside case of the appliance nonconductive. Such appliances are called *double-insulated*, since the insulating case serves as a second layer of insulation above and beyond that of the conductors themselves. If a wire inside the appliance accidently comes in contact with the case, there is no danger presented to the user of the appliance.



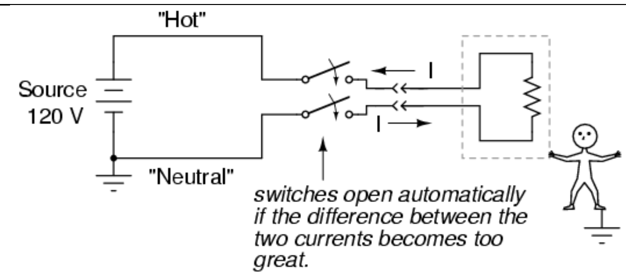

**Chassis Grounding Alone, Pro and Con**: Other engineers tackle the problem of safety by maintaining a conductive case, but using a third conductor to firmly connect that case to ground. [This is the green or bare metal ground conductor added in the 1960s.]

The third prong on the power cord provides a direct electrical connection from the appliance case to earth ground, making the two points electrically common with each other. If they're electrically common, then there cannot be any voltage drop between them [not entirely, see below]...If the hot conductor accidently touches the metal appliance case, it will create a direct short-circuit back to the voltage source through the ground wire, tripping any overcurrent protection devices.

According to the textbook *EEAI3* p. 295-7, however, there is still some current through the man in this case, and it is still possibly hazardous, especially if the grounding is faulty. In addition, the circuit breaker does not usually trip, so the problem persists.




**Ground Fault Circuit Interrupter GFCI, Pro**: When a toaster has a faulty hot connection to the metal case, a person may be shocked by providing an alternate path to ground. In this case, the hot and neutral currents are no longer exactly equal, being greater on the hot side




However, by interposing a *Ground Fault Current Interruptor*, or GFCI, the difference in current comparing hot and neutral is readily detected, and the GFCI shuts off current to the faulty toaster very rapidly. These are often used in wet areas such as bathrooms and kitchens. (See textbook p. 306, and diagram further below.)

**Bonding to Neutral Alone, Con**: If instead of grounding a chassis, one *bonds* (connects) the chassis to neutral, a fault is more likely to trip the circuit breaker. The textbook finds this a fairly good approach, but it has the drawback that heavily loaded equipment causes a significant voltage on the chassis even in the absence of a fault. The textbook p. 299 gives an example in which the chassis voltage proves "high" at 9V.

**EGC Grounded Chassis Plus Bonding of Ground to Neutral, Pro**: This is one of the better solutions, and was adopted in the US and most of the world. (textbook p. 299-300). The neutral is grounded at the service panel. In addition, an Equipment Grounding Conductor EGC is grounded locally near the service panel (copper rod, etc.) and provides a ground connection to all cases and chassis that might otherwise be hot from a fault.

Using this approach, the chassis voltage under heavy load is about 3 V rather than 9 V for the example to the left, which is an improvement and apparently not considered harmful. See textbook pl 302. (A GFCI might still be nice however.)

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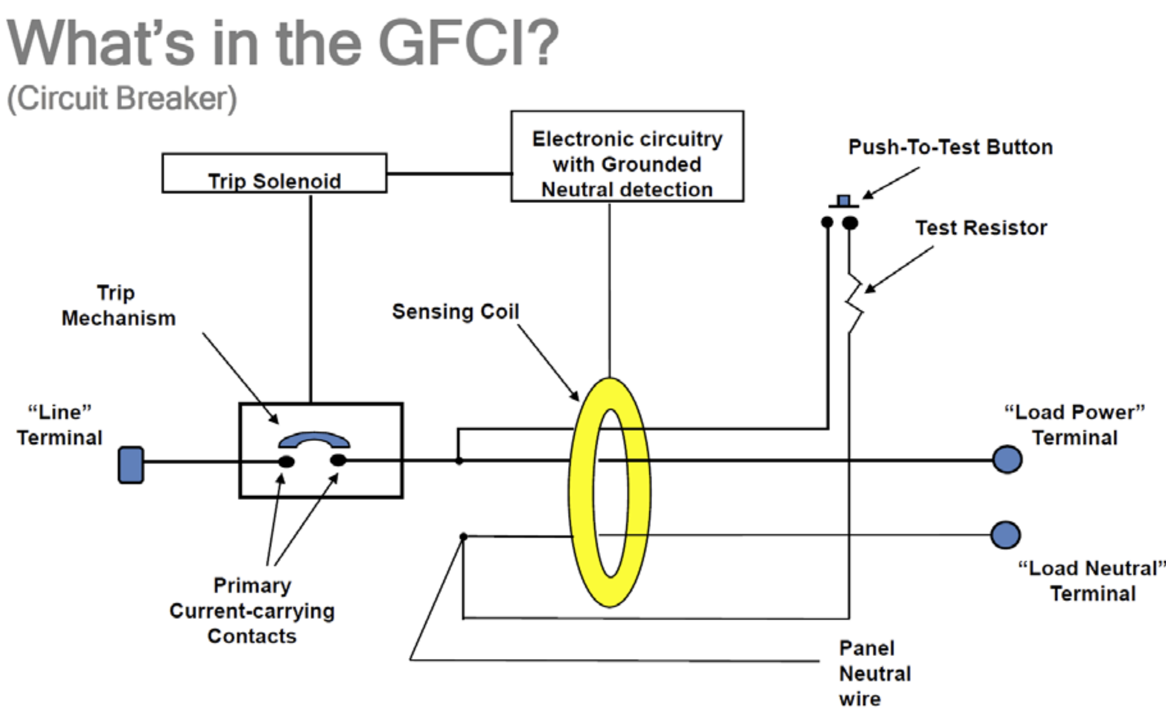
|---|
| **The Ideal Residential Electrical Service, Pro**: According to the textbook,[337] the US and most other countries failed to adopt the safest possible residential electrical service configuration. In this system, the power company provides 3 separate wires to each house: hot, neutral, and ground (EGC). Neutral is connected to the EGC only close to the transformer, so the development of a voltage on a chassis due to heavy loading does not arise (i.e., the situation discussed above as Bonding to Neutral Alone). The EGC is grounded also near the transformer, but not otherwise. An internal fault sends high current along a path to ground of very low resistance, thus easy to clear with a circuit breaker. The electric utility companies however regard the third wire to be unnecessary, or at least not providing sufficient benefit to justify the extra cost. |
| **Broken Shared Neutral Wire (Loss of Neutral Integrity), Con**: The textbook p. 306-7 describes an unusual and surprising cause of a shock hazard. It occurs when two (or more) homes share the same hot and neutral lines from a shared distribution transformer. The hot lines are intact, but the neutral from House 1 is broken, past the point where the neutral branch to House 2 is given off. Current which would normally return from House 1 on its Neutral 1 is interrupted, so instead its current passes from neutral 1 to ground, and returns to the transformer via the ground connection for neutral at the transformer. However, even if the hot is not energized at House 2 (its power is off), if a person at House 2 touches a grounded metal case or chassis, he provides an alternate return path for the ground current leakage: the current partially passes from ground through his body to the chassis, then via the EGC to the bonded neutral for house 2, with final return on the intact part of the neutral to the transformer. In summary, a fault affecting House 1 wiring can lead to a potentially lethal shock hazard for a person touching a grounded chassis in House 2. How truly bizarre! |

More About How a Ground Fault Circuit Interrupter GFCI works




The working of a GFCI circuit breaker is sketched in this NEMA diagram above.[338] The current on the ungrounded hot "load power" line should be exactly equal to the current returning on the load neutral wire. In the sensing coil, the two currents coil in opposite winding directions. This normal condition results in no net magnetic flux generated in the sensing coil. If there instead is a fault current, the load neutral and load

[337] *EEAI3* p. 309-10
[338] https://www.nema.org/Products/Documents/NEMA-GFCI-2012-Field-Representative-Presentation.pdf

power currents passing through the sensing coil are not exactly equal, the sensing coil detects this imbalance (as little as 4 to 6 mA greater current in the hot "load power" line). This causes the trip solenoid to open the trip mechanism, shutting off the hot "load power" line and apparently also the neutral line.[339] Pushing the Test button introduces a small current diversion (with current limited by the test resistor), creating an imbalance sensed at the sensing coil, opening the trip mechanism. The Reset button restores the closed status of the trip mechanism.

Note: "All GFCI outlets have one little-known flaw: their circuitry eventually wears out, usually after about 10 years... The reset button alone won't tell you if a pre-2006 GFCI outlet is still working properly—you'll need to check it with a special tester... All GFCIs manufactured after mid-2006 are designed to tell you when they fail. The vast majority indicate failure by shutting off power permanently. So someday your GFCI (and any other outlets connected to it) will simply stop delivering power and you'll have to replace it."[340]

GFCE devices should be tested at regular intervals to be sure they are still working.

# Power Quality

We skipped this important but advanced subject in our class, discussed in Chapters 13,[341] and I offer only some highpoints.

Problems can arise when two or more loads arranged in parallel share the same voltage source. This arrangement lowers the total impedance Z across the multiple loads, and this leads to reduction of the load voltage.

## Types of Load Voltage Fluctuations

Variations of the load voltage take on several possible forms:

- *Fluctuations*: small changes between 90% and 110% of the rated value. If slow and infrequently recurring (over hours, with frequency ≪ 60 Hz), it may not be noticeable by people or equipment

- *Flickers*: These are fast and sometimes cyclic change in V that are often readily detected visually and can be annoying. They can also cause computer freezeups, jitter of TV images, faulty signals and malfunctions in electronics, and loss of stored information, etc.. The rapidity of change makes it more apparent. The textbook states that incandescent lights are more susceptible to visible flicker (19% reduction in light intensity in the book example for a 10% reduction in voltage) compared to ballasted fluorescent lighting (10% reduction in light intensity in the same example). This is attributable to differences in *gain factor*,[342] a measure of how much the light intensity changes when the voltage fluctuates. Flicker can be reduced by installing a series capacitor on the distribution feeder. (In the textbook example, the magnitude of load voltage reduction when a 2nd load is switched on is reduced from 12.5% to only 0.1%.)

  Humans detect flicker with greatest sensitivity (and become annoyed maximally) at a flicker recurrence rate of about 8-10 Hz, whereas flicker is visually undetectable beyond a *critical flicker frequency CFF* of as low as 35 Hz for many humans, but up to 80 Hz or more for the most sensitive folks. Flicker may still be harmful to equipment even if not visually apparent. Flicker is more apparent when it is of high amplitude, when it is recurrent, when mean light intensity is greater, and also when the light wavelength (spectral composition) falls in the range where human perceptibility of light is maximal (a relationship expressed by its *luminous flux*). Other factors also enter in.[343]

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339 http://www.elec-toolbox.com/Safety/safety.htm#GFCIs

340 http://www.familyhandyman.com/electrical/wiring-outlets/testing-gfci-outlets/view-all

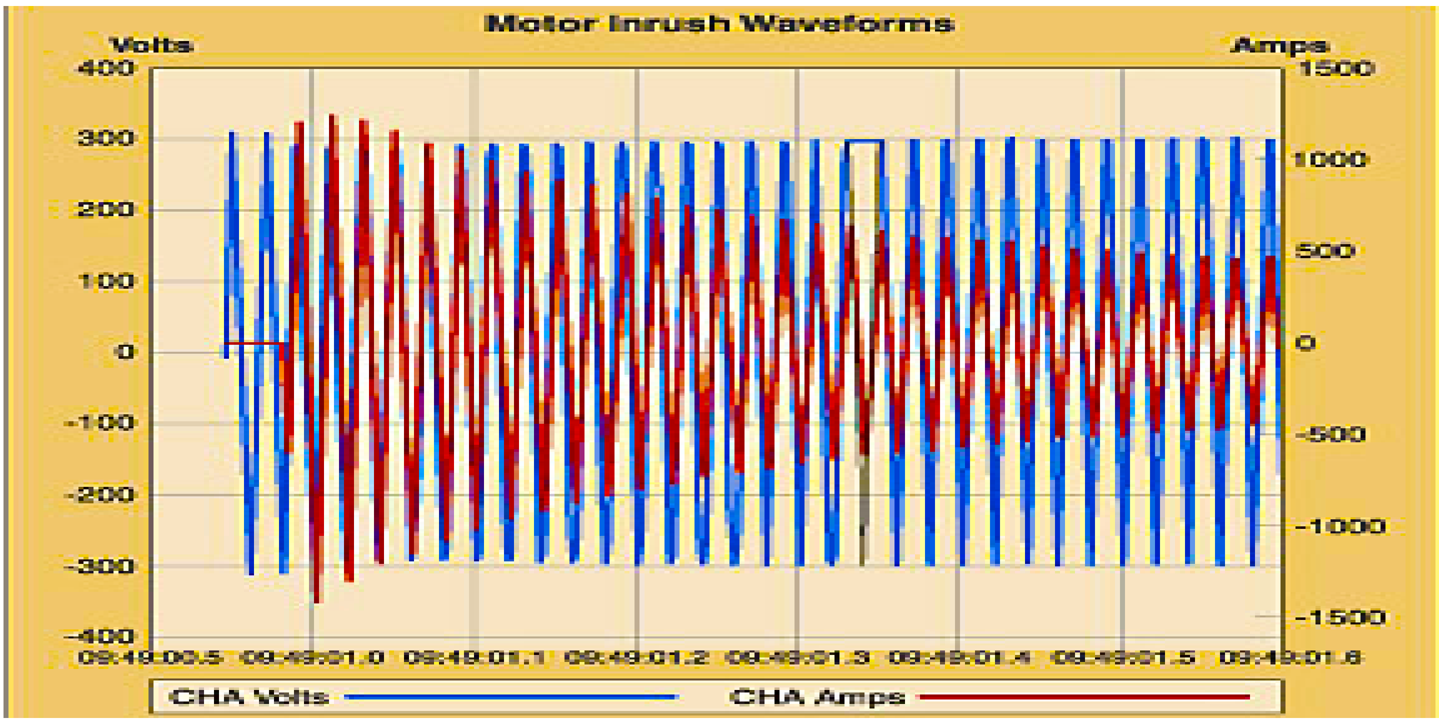
341 *EEAI3* p. 418-513

342 http://www.ccohs.ca/oshanswers/ergonomics/lighting_flicker.html Note that this article states that flicker is less with electronically ballasted fluorescents [which output rapid AC of over 20 kHz], but that incandescent bulbs flicker less than old-fashioned magnetically ballasted fluorescents. This topic seems to generate active debate.

343 http://webvision.med.utah.edu/book/part-viii-gabac-receptors/temporal-resolution/ and http://home.ieis.tue.nl/rcuijper/reports/Perz%2520M_Master%2520Thesis_Flicker%2520Perception%2520in%2520the%2520Periphery.pdf and https://en.wikipedia.org/wiki/Flicker_fusion_threshold and https://en.wikipedia.org/wiki/Luminous_flux

- *Sags*: Voltage drops below 90% of rated for up to a few seconds. These are common power quality problems. They occur normally when heavy electric loads switch on, such as elevators, pumps, A/C compressors and heating blowers, refrigerators, vacuum cleaners, etc. When motors and transformers are first energized, the *inrush current* can be high (as much as 25 times the full-load steady state rated current), and these commonly cause voltage sags. (It may be necessary to add an inrush current limited circuit to prevent the high current from tripping circuit breakers or causing other damage.) Sags of course can also arise from faults, such as a tree falling across a line, which acts like an impedance added in parallel, thus reducing the load voltage. The impact of the fault is greater when the Voltage sag VS can be expressed by:
  $$VS \text{ (Voltage Sag)} = (V_{load} - V_{ss})/V_{ss}$$
  and may take on negative or positive values. The textbook gives an example where switching in a motor causes a VS = −24% due to the inrush current. This is reduced by adding in a capacitor in parallel to the load (motor)—specifically, adding a 6 Ω capacitor reactance changes the VS to +3%.[344]
- *Swells*: Voltage rises above 110% of rated for up to a few seconds.
- *Undervoltage (Brownout)*: Voltage below 90% for at least several minutes.
- *Overvoltage*: Voltage above 110% for at least several minutes.
- *Interruptions*: decrease in voltage below 10% of rated for any period.

An example of the inrush current in one of the 3 phases of a motor is illustrated (below).[345] Here, the voltage is in blue, the superimposed current in red. The time scale at the bottom shows that the current begins to




flow in the first division on the left, just before 09:49:01.0, at which time the current rapidly builds up to a $I_{rms}$ peak of about 850 A (by 09:49:01.0, value estimated from a separate graph not shown). The current then tapers off to the equilibrium value which has been nearly attained at the right side of the graph, around 180 A rms at about 09:49:04.0 (i.e., 3 seconds after the start, also shown on a separate graph). During the peak of the inrush current, the voltage shows a sag, from ~220 V rms to ~ 206 V, or about 14 V rms, then rises to a new slightly lower steady-state value reflecting the load that has been added.

The transient inrush current peak decays to the steady state with an exponential decay factor or factors.

Motor inrush currents are high in high-efficiency motors.

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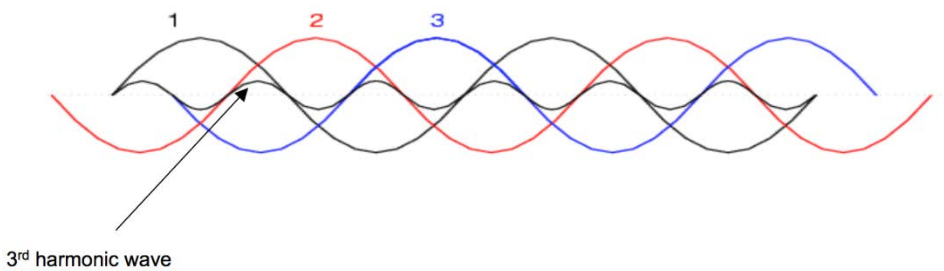
[344] *EEAI3* p. 492.

[345] http://ecmweb.com/site-files/ecmweb.com/files/archive/ecmweb.com/mag/612ecmPQfig2.jpg , image slightly modified by MCM and stretched horizontally.

## Harmonic Distortion

Periodic waveforms that are not purely sinusoidal can be represented by a Fourier Series (details omitted).[346] These include a series of terms involving sin(kωt) and current components $I_k$, where k has values 0,1.2....The *Individual harmonic distortion IHD* for a component with index k and angular frequency kωt is the fraction $I_k/I_1$. Here, $I_1$ is the current in pure sinusoidal form for the fundamental frequency. The *Total harmonic distortion THD* is $I_H/I_1$ , where $I_H = \sqrt{I_{dc}^2 + I_2^2 + I_3^2 + \cdots}$ Thus the THD is a measure of the total distortion of the waveform arising from DC and harmonics compared to a purely sinusoidal fundamental signal $I_1$ having frequency f=2πω = 1/T (in Hz or $s^{-1}$). Here, T= waveform period =1/f = 1/2πω (in s). THD is ≥ 0 and can exceed 100%.

An example of a problem arising from THD in a three phase system: the three phases (shown as 1, 2, and 3 in the diagram) will theoretically cancel each other out at the neutral wire. However, if the 3 phases contain 3rd order harmonics, the currents will not fully add to zero. As seen in the figure, the 3rd harmonics from the 3 phases will lead to an oscillating current in the neutral wire ("3rd harmonic wave"), which can be dangerous since the neutral is expected and designed to carry minimal current...[347]




Large and/or non-linear loads such as arc furnaces (aluminum smelters), adjustable speed drives (i.e., *variable frequency drives VFDs*),[348] and power electronic (such as SCR converters) can produce significant harmonic distortions in the bus voltage and power grid.[349] The THD of computer power supplies, monitors, AC/DC converters, electronic ballasts, X-ray and MRI equipment, and Uninterruptible Power Supply UPS power can be very large, again apparently because they represent non-linear loads.[350] The textbook offers these ranges of THD in load currents by types of load:

Fluorescent Lamp THD=15-25%
Adjustable Speed Drives VFDs THD=30-80%
Personal Computers THD=70-120%
Computer Monitors THD = 60-120%.

Although individual computers and monitors draw low currents, they often are present in large numbers on dedicated feeders, causing the feeder to be "highly polluted with harmonics". A better strategy is to intersperse these nonlinear devices with simpler loads such as heaters and lights. The textbook gives an example in which a polluted feeder feeding many computers having THD=65% is "treated" by adding a heating load, resulting in a reduction of THD to 17%.[351]

In summary, harmonic distortion THD can cause many serious problems:

- resonances that can produce very high voltages that are potentially dangerous or damaging to equipment

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[346] https://en.wikipedia.org/wiki/Fourier_series and
https://en.wikipedia.org/wiki/Harmonics_%28electrical_power%29
[347] Image (by author Dkangell) copied and text paraphrased from
https://en.wikipedia.org/wiki/Harmonics_%28electrical_power%29#/media/File:3rd_orderHarmonics.png
see also http://www.pge.com/includes/docs/pdfs/mybusiness/customerservice/energystatus/powerquality/harmonics.pdf
[348] https://en.wikipedia.org/wiki/Variable-frequency_drive
[349] *EEAI3* p. 501
[350] http://www.eweb.org/powerquality/harmonic and other sources
[351] *EEAI3* p. 502

- increased losses in transmission lines, transformers, and generators; possibility of resonance damage to insulators and lines from high voltage
- damage to capacitor banks (from overvoltage)
- overvoltage or added drag forces (*drag torque*) in synchronous motors. (The textbook p. 509 gives an example of a 3-phase motor for which the normal rotation sequence of the magnetic field of the fundamental is a-b-c, but for which the 5th harmonic exerts an out-of-sequence a-c-b drag torque opposing the fundamental.) This can lead to sluggish rotation.
- reduction of the overall power factor pf. Only voltages and currents of the same frequency produce real power. Conventional low-cost devices used to measure power output and power factors are inaccurate in the presence of severe harmonics. More accurate but much more expensive meters can be justified by the power company wanting to maximize billable kWh.
- increased EM interference of communication networks
- premature aging of insulators
- picture jitters; freezing and rebooting of computers and other sensitive equipment such as cell phones, due to magnetically-induced voltages, etc.

This is clearly a complex and important subject.

## Power Grid and Blackouts

We minimally discussed these topics, presented in Chapter 14,[352] and I will take time only to offer a few points.

Power companies choose generally to build generating capacity to meet the average regional power needs. They find that it is not economically feasible to build capacity to handle the highest possible demand encountered. Instead, the grid is interconnected, and companies buy power to meet high excess demand, and sell power when generation exceeds local demand.

A blackout may result from a deficiency or a surplus in power relative to power demand exists, if not corrected in seconds—a *power balance* must be maintained. Some bulky systems (e.g., hydroelectric turbines) have relatively long lag times in responding to requests to change output.

When demand is reduced, must slow generators by reducing water input (can take 7 to 10 s), and in thermal plants, can allow steam to escape and reduce combustion, etc. Can also increase exports and/or decrease imports.

When demand is increased, can increase water input to turbines, recruit already *spinning reserves* if available (these are rotating generators on standby that are not yet generating). Can also decrease exports or increase imports.

Greater interconnectedness provides more opportunities to import needed power, but also increases complexity and makes it possible for blackouts to spread to wider areas. If power generation is insufficient for local demand, we may reduce exports, increase generation rate where possible, reduce loads by disconnecting some where feasible (*rolling blackouts*), and import more power.

Topology of the grid is important, in terms of how various power stations are interconnected for maximizing stability while minimizing cost of transmission lines and control equipment. Detailed analysis requires solving nonlinear equations of great complexity.

Electrical demand in an urban region like Seattle tends to exhibit two peaks of demand, one around 9 AM and the other around 6 PM.[353] Demand above the generating capacity prompts importing of power from neighboring utilities, whereas demand below generating capacity prompts exporting of power to neighboring utilities (or reduction of production). The Northwest imports power from the East at noon ET and exports to the East at 9 AM ET. In summer, the NW exports electricity southward for air conditioning, whereas it imports it during the winter heating months.

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[352] *EEAI3* p. 515-539
[353] An empiric formula said to be for the Seattle-area demand is given on *EEAI3* p. 524.

## North American Interconnections and Reliability Councils

In North America, there is a web of interconnecting utilities allied under the umbrella non-profit organization, *North American Electric Reliability Corporation NERC*,[354] formed to "promote the reliability and adequacy of bulk power transmission in the electric utility systems of North America." It oversees:

The *Eastern Interconnection* (covers most of eastern North America, extending from the foot of the Rocky Mountains to the Atlantic seaboard, excluding most of Texas):

- Florida Reliability Coordinating Council (FRCC)
- Midwest Reliability Organization (MRO)
- Northeast Power Coordinating Council (NPCC)
- ReliabilityFirst (RF ). This is the successor to three reliability organizations: the Mid-Atlantic Area Council (MAAC), the East Central Area Coordination Agreement (ECAR), and the Mid-American Interconnected Network (MAIN)
- SERC Reliability Corporation (SERC)
- Southwest Power Pool, Inc. (SPP)

The *Western Interconnection* covers most of western North America, from the Rocky Mountains to the Pacific coast. It is tied to the Eastern Interconnection at six points, and also has ties to non-NERC systems in northern Canada and Northwestern Mexico. Its reliability council is:

- Western Electricity Coordinating Council (WECC)

There are also:

The *Texas Interconnection*

- Texas Reliability Entity (TRE)

The *Quebec Interconnection*

The *Alaska Interconnection*, for which the reliability council is

- Alaska Systems Coordinating Council (ASCC),

Maps of much of the US grid are shown in the section, "Electrical Transmission". Controls on these various systems are loose and diverse. Recent 2014 *Zonal Topology Diagrams* of the WECC for summer and winter are given here.[355]

## Causes of Blackouts

See also discussion in Future Power Systems.

Blackouts are more likely to occur in times of heavy loading, when plants are generating near their limits and unused reserve capacities (including spinning reserves are minimal. Blackouts can occur due to

- Faults in transmission lines causing excessive currents
- Lightning (causing insulator failure), earthquake (damaging substations), strong winds (felling trees), heavy frost and ice storms (breaking heavily laden lines)[356]
- Failure of major devices, such as generators and transformers. For instance, a synchronous generator can quickly go out of synchronism when mechanical input power and output demand are not in balance.
- Improper function of protection and control devices
- Breaks in communication links
- Human errors

The textbook lists major US blackouts:

- 1965: On the evening of November 9, the Northeast blackout of 1965 affected portions of seven northeastern states in the United States and the province of Ontario in Canada. It affected 30M

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[354] https://en.wikipedia.org/wiki/North_American_Electric_Reliability_Corporation
[355] https://www.wecc.biz/Reliability/2014PSA_draft.pdf
[356] https://en.wikipedia.org/wiki/Ice_storm

people and took 24 hours to restore the system. It began with tripping of an improperly set relay protecting a line.
- 1977: On July 13–14, the New York City blackout of 1977 resulted in looting and rioting. It affected 8M people and lasted 25 hours. It began with lightning.
- 2003: On August 14, the Northeast blackout of 2003, a wide-area power failure in the northeastern USA and central Canada, affected over 55 million people. This lasted several days. It began with a coal-generation plant going off-line, tripping of an overloaded line, etc.

Severe global blackouts, affecting more than 100M people, include:
- 2012: July 30 and 31, 2012 India blackouts, the first affected 300M people, the second affected 620M people (the largest in history), the two lasted 2 days. Began with tripped breakers during a time of above normal power demand
- 2001: January 2, 2001, India, affected 230 M people
- 2014 November 1, 2014, Bangladesh blackout, affected 150M
- 2015 January 26, 2015, Pakistan blackout, affected 140M.
- 2005: August 18, 2005, Java-Bali blackout, affected 100 M.

## Future Power Systems

We did not take time for this subject, discussed in Chapter 15.[357] Some of the topics of interest include:

- Smart Grid[358], including some of the following components
    - Improved accessibility and bidirectional flow[359] (e.g., from customer PV)
    - Improved system flexibility of network topology, *Combined Heat and Power* (CHP) Systems[360]
    - Increased system capacity
    - Improved reliability, reduced blackouts and forced outages
    - Renewables (large scale and customer owned) integrated with weather prediction and operated with greater central communication and control[361]
    - Improved Methods of Storing energy (*Energy Storage Systems ESS*, such as *Superconducting Magnetic Energy Storage SMES*,[362] Batteries, Hydrogen $H_2$ combined with Fuel Cells)
    - *Intelligent Monitoring* with advanced meters and sensors, rapid data-intensive analysis and rapid secure communications, Geosynchronous and Low Earth orbit satellites, *Phasor Measurement Units PMU* aka *Synchrophasors* (used to synchronize phase among remotely connected buses based on a single clock). The real and reactive power flowing from bus 1 to a widely separated second bus 2, where the power angle = δ and the inductive reactance of the line = x, are given by[363]
    - $P_2 = \frac{V_1 V_2}{x} \sin\delta$ and $Q = \frac{V_2}{x}(V_2 - V_1 \cos\delta)$

      provided these measurements are made with precisely synchronized time using PMUs.
    - Peak demand shaving, grid-friendly load adjustment (load tracking) and balancing
    - Smart House with home Energy Management System EMS, Smart Appliances (which can be remotely controlled to reduce load when needed), Home Power Generation.[364]
    - Self-Diagnosis, Fault detection, Self-Healing Grid, Rapid restoration, Substation and distribution automation
    - Sustainability

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[357] *EEAI3* p. 541-557
[358] http://energy.gov/oe/services/technology-development/smart-grid and https://en.wikipedia.org/wiki/Smart_grid
[359] http://www.nist.gov/smartgrid/beginnersguide.cfm
[360] http://energy.gov/oe/technology-development/smart-grid/distributed-energy/combined-heat-and-power-chp-systems
[361] https://www.smartgrid.gov/the_smart_grid/smart_grid.html
[362] http://www.sciencedirect.com/science/article/pii/S1875389212021645
[363] *EEAI3* p. 545
[364] https://www.smartgrid.gov/the_smart_grid/smart_home.html

  - Improved power system security and reduced vulnerability.

- *Plug-In Electric Vehicles PEV* and *Plug-In Hybrid Electric Vehicles PHEV* (with charging rates responsive to grid conditions to prevent blackouts)[365]
- Alternative Renewable Energy Resources: Wind, solar, hydrokinetic, geothermal, biomass, $H_2$, etc.
- Less Polluting Power Plants: using coal gasification syngas, carbon sequestration
- Distributed Generation Systems: including Natural Gas and/or Fuel Cell based home electricity generation
- Improved Power Electronics
- Enhanced Reliability of Power Systems
- Intelligent Operation, Maintenance , and Training
  - Robots
  - Virtual Monitoring, Virtual reality techniques
- Space-Based Power Plants?

[365] https://www.smartgrid.gov/the_smart_grid/plugin_electric_vehicles.html

# Glossary and Mini-Topics

Topics and terms are included here for added emphasis or for when they are not fully treated in the body of this summary.

**Dynamo vs. Alternator**

The word dynamo became associated exclusively with the commutated direct current electric generator, while an AC electrical generator using either slip rings or rotor magnets would become known as an **alternator**.[366]

**Volt-Ampere VA, Watts, and UPS Selection:**

"A volt-ampere (VA) is the unit used for the apparent power in an electrical circuit, equal to the product of root-mean-square (RMS) voltage and RMS current... Apparent power is the magnitude of the vector sum (S) of real (P) and reactive (jQ) AC power vectors... Some devices, including uninterruptible power supplies (UPSs), have ratings both for maximum volt-amperes and maximum watts. The VA rating is limited by the maximum permissible current, and the watt rating by the power-handling capacity of the device. When a UPS powers equipment which presents a reactive load with a low power factor, neither limit may safely be exceeded. For example, a (large) UPS system rated to deliver 400,000 volt-amperes at 220 volts can deliver a current of 1818 amperes... In direct current (DC) circuits, this product is equal to the real power (active power) in watts. "[367]

Neil Rasmussen/APC: "The power drawn by computing equipment is expressed in Watts or Volt-Amps (VA). The power in Watts is the real power drawn by the equipment. Volt-Amps is called the "apparent power" and is the product of the voltage applied to the equipment times the current drawn by the equipment. Both Watt and VA ratings have a use and purpose. The Watt rating determines the actual power purchased from the utility company and the heat loading generated by the equipment. The VA rating is used for sizing wiring and circuit breakers.

The VA and Watt ratings for some types of electrical loads, like incandescent light bulbs, are identical. However, for computer equipment the Watt and VA ratings can differ significantly, with the VA rating always being equal to or larger than the Watt rating. The ratio of the Watt to VA rating is called the "Power Factor" and is expressed either as a number (i.e. 0.7) or a percentage (i.e. 70%).

UPS have both Watt ratings and VA ratings. Neither the Watt nor the VA rating of a UPS may be exceeded. In most cases, UPS manufacturers only publish the VA rating of the UPS. However, *it is a standard in the industry that the Watt rating is approximately 60% of the VA rating, this being the typical power factor of common loads*. Therefore, it is safe to assume that the Watt rating of the UPS is 60% of the published VA rating.

Using APC sizing guidelines or an APC Configuration can help avoid these problems, as the load power values are verified. Equipment nameplate ratings are often in VA, which makes it difficult to know the Watt ratings. If using equipment nameplate ratings for sizing, a user might configure a system which appears to be correctly sized based on VA ratings but actually exceeds the UPS Watt rating.

By sizing the VA rating of a load to be no greater than 60% of the VA rating of the UPS, it is impossible to exceed the Watt rating of the UPS. Therefore, unless you have high certainty of the Watt ratings of the loads, the safest approach is to *keep the sum of the load nameplate ratings [in watts or V-A] below 60% of the UPS VA rating*. Note that this conservative sizing approach will typically give rise to an oversized UPS and a larger [backup] run time than expected... "[368]

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[366] https://en.wikipedia.org/wiki/Dynamo
[367] https://en.wikipedia.org/wiki/Volt-ampere
[368] http://www.apcmedia.com/salestools/SADE-5TNQYF/SADE-5TNQYF_R1_EN.pdf